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Study of Soliton Interaction in Optical Fibers with Third Order Dispersion and Higher Order Nonlinear Effects

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***Abstract*—In this paper, we present a numerical approach to solve the GNLSE and analyze soliton interaction phenomena using COMSOL environment. By leveraging the capabilities of COMSOL's PDE module, we can accurately capture the dynamics of solitons and investigate their interactions. We analyze the impact of different parameters such as soliton power, initial separation distance, and dispersion characteristics on the soliton dynamics. Furthermore, we examine the role of higher-order dispersion terms in shaping the soliton interactions. Our findings demonstrate the effectiveness of the proposed numerical approach in accurately simulating and analyzing soliton interaction phenomena. The COMSOL-based methodology provides a flexible and efficient framework for studying complex nonlinear optical systems, enabling researchers to gain insights into the behavior of solitons in different media and design optimized communication systems. This paper contributes to the understanding of soliton dynamics and provides a practical tool for investigating the behavior of solitons in nonlinear dispersive media. The presented numerical approach using COMSOL opens avenues for further research in nonlinear optics and fiber optic communication systems**.

***Keywords*— FEM, higher order nonlinearity, soliton interaction, optical fibers**.

I. Introduction

Fiber solitons are fascinating phenomena that occur in optical fibers, which are long, thin strands of glass or plastic used to transmit information in The form of light pulses. Solitons are self-sustaining, localized wave packets that retain their shape and velocity as they propagate through a medium [1]. In the context of fiber optics, fiber solitons are special types of optical solitons that form due to a delicate balance between the dispersive and nonlinear properties of the fiber [2]. Dispersion refers to the spreading out or broadening of an optical pulse as it travels through the fiber, while nonlinearity represents the dependence of the fiber's refractive index on the intensity of the light passing through it [3]. When these two opposing effects interact in the right manner, a fiber soliton can be generated. The nonlinear nature of the fiber causes the light pulse to self-focus, compensating for the dispersive effects and preventing the pulse from dispersing or spreading out over long distances [4,5]. This ability to maintain its shape and size enables the soliton to travel long distances without significant distortion. [6].

Stimulated Raman scattering (SRS) is a nonlinear optical process that occurs in fiber due to the interaction between the intense optical pulse and the molecular vibrations of the fiber material [4]. It leads to the transfer of energy from the soliton to lower-frequency components through the generation of new frequencies via Raman amplification. As a result, the soliton experiences energy loss and spectral broadening. SRS can limit the distance over which solitons can propagate without significant degradation, and it becomes more pronounced as the power or duration of the soliton increases [7]. Self-steepening (SS) is a phenomenon that arises from the nonlinear nature of the fiber medium. It causes the leading edge of the soliton to propagate faster than the trailing edge, resulting in a steepening of the pulse shape during propagation [8]. This effect leads to spectral broadening and an increase in the peak power of the soliton [4]. SS

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can affect the stability and duration of the soliton, and it becomes more prominent for shorter duration pulses. Dispersion is the phenomenon where different spectral components of an optical pulse travel at different speeds, causing pulse broadening. Third-order dispersion (TOD) refers to the variation of the dispersion with wavelength [2,4]. In the context of fiber solitons, it can introduce temporal oscillations and affect the pulse duration and shape. TOD can counteract the self-focusing effect of the fiber nonlinearity, making it more challenging to maintain soliton propagation over long distances [2]. Compensation techniques, such as dispersion management, can be employed to mitigate the adverse effects of TOD [9]. It's important to note that these effects do not completely destroy the soliton's properties, but they can modify its characteristics and impose limitations on its propagation distance and stability. Researchers and engineers working with fiber solitons need to carefully consider and manage these effects to optimize soliton-based communication systems and ensure reliable data transmission over long distances [6,7].

In this paper, we will solve the generalized propagation equation involving SRS, SS, and TOD effects using the COMSOL environment and we will highlight the evolution of the soliton and the interaction of the soliton and show the effects of the soliton order.

II. Generalized Nonlinear Schrodinger Equation

The Generalized Nonlinear Schrödinger Equation (GNLSE) is a theoretical description used to study the propagation of optical pulses in nonlinear optical fibers. It is an extension of the Nonlinear Schrödinger Equation (NLSE) that takes into account additional effects such as higher-order dispersion, SS and SRS. The GNLSE can be written as [3,4,10-13]

$$\frac{\partial A}{\partial z} - \overbrace{\underbrace{j\sum_{n=2}^{\infty}\frac{j^n\beta_n}{n!}\frac{\partial^n A}{\partial T^n}}_{Dispersion} + \underbrace{\frac{\alpha}{2}A}_{Loss}}^{Linear\ effect} = j\gamma\overbrace{\left(\underbrace{A|A|^2}_{SPM} - \underbrace{\tau_R A\frac{\partial |A|^2}{\partial T}}_{Raman\ effect} + j\underbrace{\tau_s\frac{\partial\left(A|A|^2\right)}{\partial T}}_{self-steepening}\right)}^{Nonlinear\ effect} \quad (1)$$

where $A(z,T)$ is the slowly varying electric field envelope of the optical pulse, z is the propagation distance, T is the pulse time, β_n represents the nth-order dispersion coefficient, the parameter α represents the fiber attenuation, γ is the nonlinearity coefficient, $\tau_s = 1/w_o$ and τ_R is the characteristic time for the SRS. The parameters β_2 stands for group velocity dispersion (GVD) and the higher-order dispersion terms (β_3, β_4 , etc.) capture the effects of group velocity dispersion beyond the second order. The right-hand side models nonlinear effects, where the first term on the right represents the Kerr nonlinearity, which describes the intensity-dependent refractive index of the medium. It leads to self-phase modulation (SPM) and self-focusing or self-defocusing effects, depending on the sign of nonlinearity parameter. The first time derivative term represents the dispersion of the nonlinearity, i.e. SRS, while the second derivative term usually associated with effects such as SS and optical shock formation [14,15].

This equation was solved numerically, and it is the required to study the soliton in optical fibers. Numerical methods such as split-step Fourier methods or finite-difference methods are commonly used to solve the GNLSE and obtain the pulse evolution in nonlinear media. These well-known methods have drawbacks such as the long period of calculations and the associated inaccuracy. In this research, we prepared a COMSOL environment to solve the equation, which is characterized by fast completion and higher accuracy. For pulses of width $T_\circ > 5ps$, the parameters $(w_o T_o)^{-1}$ and $\tau_R / T_\circ$ become so small, such that the last two terms in Eq.(1) can be neglected. As the contribution of the third order dispersion is also quite small for such pulses one can employ the reduced NLSE [1,16].

III. The Normalized GNLSE

When discussing the soliton problem, it is best to work on a normalized propagation equation. In order to obtain this equation that is without units, we will perform the following transformations [4,17]

$$u(\zeta,\tau) = \frac{A}{\sqrt{P_o}} \ , \ \zeta = \frac{z}{L_D}, \quad \tau = \frac{T}{T_o} \ , \ L_D = \frac{T_o^2}{|\beta_2|}, \ L_{NL} = \frac{1}{\gamma P_o}$$

onto Eq.(1), keeping only the second and third terms of dispersion, and neglecting the attenuation, to get [3]

$$\frac{\partial u}{\partial \zeta} - i\frac{sgn(\beta_2)}{2}\frac{\partial^2 u}{\partial \tau^2} - \delta_3\frac{\partial^3 u}{\partial \tau^3} = iN^2\left[|u|^2 u - s\frac{\partial |u|^2 u}{\partial \tau} - iT_R u\frac{\partial |u|^2}{\partial \tau}\right] \quad (2)$$

where $P_\circ$ is the initial power, L_D is the dispersive length, L_{NL} is the nonlinear length, T_o is the pulse width and $sgn(\beta_2) = 1$ for normal GVD. The parameters

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_o T_o^2}{|\beta_2|}, \quad \delta_3 = \frac{\beta_3}{6|\beta_2|T_o}, \quad s = \frac{2}{\omega_o T_o}$$

govern, respectively, soliton order, the effects of TOD and SS. The symbol $T_R = \tau_R / T_o$ stands for intrapulse SRS [18]. All three parameters δ_3, s, T_R vary inversely with pulse width and are negligible for $T_o \gg 1ps$ but they become appreciable for femtosecond pulses [13]. The GNLSE describes both bright solitons and dark solitons, where $\mathrm{sgn}(\beta_2)$ is the sign GVD that can obtain bright and dark soliton by changing the sign. Bright soliton corresponds to the solutions of Eq.(2) with $\mathrm{sgn}(\beta_2) = 1$ and occur in normal GVD region of fibers. Similar dark solitons, correspond to solution of Eq.(2) with $\mathrm{sgn}(\beta_2) = -1$ and occur in the anomalous GVD region of fibers. The main difference compared with case of bright solitons is that $u(\tau)$ becomes a constant (rather than being zero as $|\tau| \to \infty$) [19,20]. In general, the mathematical form of the hyperbolic secant pulse is [4]

$$u(0,\tau)=\mathrm{sec}h\left(\tau\right)\left[-\frac{iC}{2}\tau\right] \quad (3)$$

where *C* is the initial chirp, $\tau = T / T_o$ and $T_\circ$ is the half width (at $1/e$ intensity point). $T_\circ$ is often replaced with the full width at half maximum (FWHM), where $T_{FWHM} = 1.763 T_o$ [2,3].

IV. SOLITON INTERACTION

The time interval between two neighboring pulses sets the bit rate of a communication system. It is thus important to determine how close two solitons can come without affecting each other. Interaction between two solitons has been studied numerically. It is clear on physical grounds that two solitons would being to affect each other only when they are close enough that their tails overlap. Numerical solutions of the GNLSE are quite instructive and allow exploration of different amplitudes and different phases associated with a soliton pair by using the following form at the input end of the fiber [1,5]

$$u(0,\tau)=\mathrm{sech}(\tau+q)+r\ \mathrm{sech}[r(\tau-q)]\mathrm{e}^{i\phi} \quad (4)$$

where *r* is the relative amplitude, ϕ is the initial phase difference and $2q$ is the initial separation between the two solitons. There are different types of solitons interactions. Soliton collision happened between two hyperbolic secant fields to collide both in linear medium and interesting nonlinear medium (soliton collision), soliton attraction will be happening in which two hyperbolic secant fields as before, but with nonlinear phase. In other, the hyperbolic secant pulses are propagated parallel to each other soliton repulsion is implemented by making one of two hyperbolic secant fields out of phase with respect to other [20].

V. RESULTS AND DISCUSSION

The soliton phenomenon is investigated based on the medium used for its transmission, as the propagation equation varies depending on the specific medium. When studying solitons through optical fibers, researchers typically examine the simplified form by considering the effects of GVD and SPM alone. Alternatively, they may also incorporate the influences of stimulated Raman scattering (SRS) and self-steepening (SS). In all cases, the soliton order is the predominant factor, determined by several variables: initial pulse width, GVD, initial power, and nonlinearity factor. In the generalized propagation equation, there exist additional influential factors that affect solitons, namely: initial chirp, SRS, SS and TOD. These factors come into play when the initial pulse width is on the order of 20 ps. During the study of soliton interaction, other factors are considered, such as the amplitude ratio, time interval between adjacent solitons, and phase difference between them. Even slight variations in these factors over small scales can have significant effects on soliton propagation. All simulations were performed by solving the propagation equation using the COMSOL environment based on FEM. The smoothness of the resulting figures depends on the mesh size. A smaller mesh size can lead to delays in computer processing and affect the accuracy of the results. Conversely, using a larger mesh size can have the opposite effect. To strike a balance between screen size and result accuracy, we selected an appropriate mesh size. During the simulation, we study the change of $u(\zeta,\tau)$ under the effects of: soliton order N, the normalized pulse width τ of the hyperbolic sec or Gaussian pulses, the sign of GVD, the effect of TOD δ_3, the effect of SRS T_R, the effect of SS, s, and the normalized propagation length $\zeta = z / L_D$. We will also study the effects of the initial chirp on the propagation of the soliton pulse.

Figure 1 to 4 represent soliton propagation through an optical fiber in the absence of TOD, SS, and SRS for the cases $N = 1, 2, 3$ respectively, at zero chirp, where the left subfigure indicates the soliton propagation spectrum and the right subfigure indicates the pulse shape at different distances for the hyperbolic sec input pulse case. **Figure** 1 represents the ideal case (fundamental soliton) $N = 1$. Notice that the soliton maintains its shape and intensity for any distance, or that it achieves the property of perfect balancing between nonlinearity and dispersion effects. This case is favorite in optical communication system. **Figure** 2 represents the case $N = 2$, it appears that the intensity increases to a maximum value and then decreases to a minimum value with the distance, and this is repeated periodically with distance. The pulse is compressed and its intensity increases at some distances, then it returns to the original shape again. Also, the pulse shape will suffer from distortions. This periodicity may also be useful in some optical communication system. **Figure** 3 represents the case $N = 3$, it appears that the evolution of the soliton through the optical fiber will initially lead to an increase in intensity in the center of the pulse, then the pulse splits into two parts with a decrease in intensity, then returns again to form a single pulse, and this behavior continues periodically. The shape of the pulse suffers from significant changes due to the propagation. The periodicity distance that was achieved in **Figure** 3 is less than that achieved in **Figure** 2. That is, an increase in the periodic distance means that the shape of the pulse is stable for a greater distance and vice versa. **Figure** 4 represents the case when $N = 4$. The pulse initially suffers from compression and an increase in intensity, then it splits into two identical parts that diverge with increasing distance and the shape of the pulse suffers from major distortions. For larger propagation distances, periodicity can also be achieved, which is not shown during the figure. For all **Figures** 1 to 4, we know that the interaction between nonlinearity and dispersion dictates the resulting behavior of the pulse during propagation. Since the nonlinearity increases with the increase in the input power, and in turn the increase in the soliton order, it is natural that this balance will be difficult to verify with the higher-order solitons except after larger propagation distances.

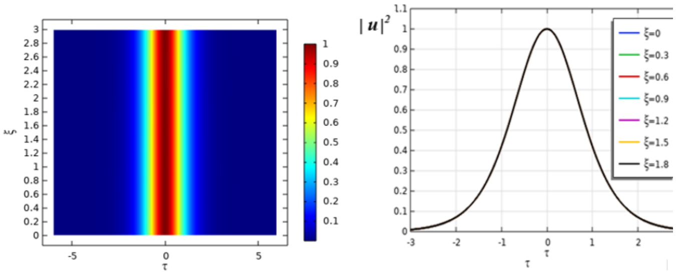

Figure 1: pulse evolution for *N*=1 soliton at zero chirp

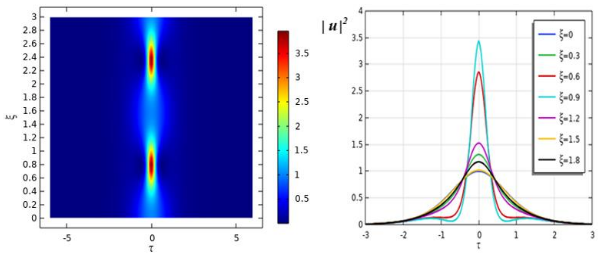

Figure 2: pulse evolution for *N*=2 soliton at zero chirp

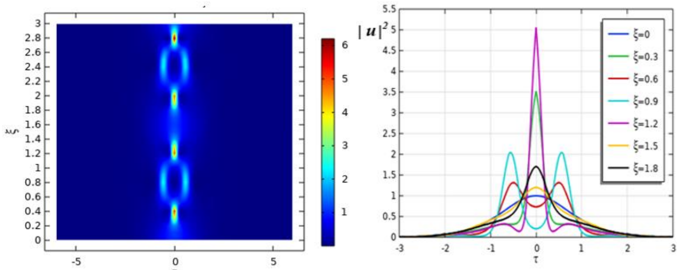

Figure 3: pulse evolution for *N*=3 soliton at zero chirp

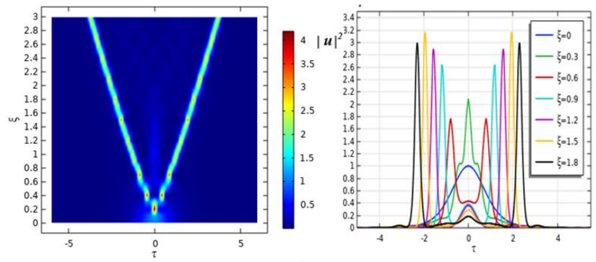

Figure 4: pulse evolution for *N*=4 soliton at zero chirp

Figures 5 and 6 obtains the soliton evolution with distance for the soliton orders $N = 1, 2$, respectively, in the absence of SS and TOD, where the left subfigure indicates the case $T_R = 0.1$ and the right subfigure refers to the case $T_R = 0.3$. **Figure** 5 shows the fundamental soliton that the effects are represented by a very small deviation to right in case $T_R = 0.1$, and a larger deviation to right and decrease in the value of the pulse intensity in case $T_R = 0.3$. These effects are not evident except with a large diffusion distance up to $\xi = 3$. **Figure** 6 represents case $N = 2$, where the amount of input power increased, and this caused a large deviation of the pulse center and a decrease in the pulse intensity in case $T_R = 0.1$. In the case $T_R = 0.3$, a large distortions and broadening of the pulse will appear, in addition to the deviation to the right. These changes occur here in a less propagation distance $\xi = 1.8$.

Figures 7 and 8 shows the pulse shape at different distances in absence the SS and TOD effects for the cases $N = 1, 2$, respectively, where the left subfigure indicates the case $s = 0.1$ and the right subfigure refers to the case $s = 0.3$. **Figure** 7 represents the fundamental soliton, in case $s = 0.1$, the peak of the pulse shows a small shift to the right that increases with distance. The reality of the case is that the displacement means an increase in the steepening of the leading edge of the pulse. In case $s = 0.3$, the steepness of the leading edge increases but the pulse base maintains the same time period. Here, it appears as if its peak has received a push to the right, which explains the physical meaning of SS. **Figure** 8 represents case $N = 2$. It appears from the figure that the irregularity is lost with the increase of the distance, as the propagated pulse suffers from shifts and distortions, in addition to the occurrence of SS. An increase in SS causes an increase in shifts and distortions. Generally, the periodicity loses in all cases.

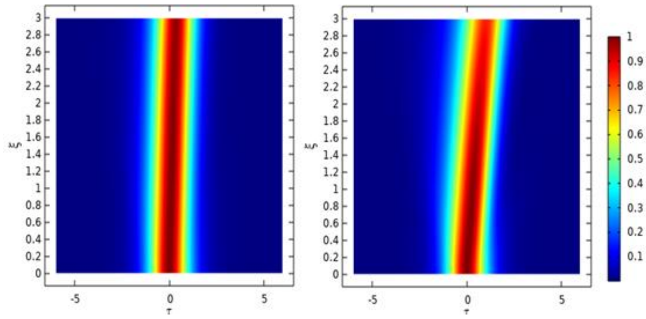

Figure 5: pulse evolution for *N*=1 soliton at $s = 0, \delta_3 = 0$ and $T_R = 0.1, 0.3$, respectively

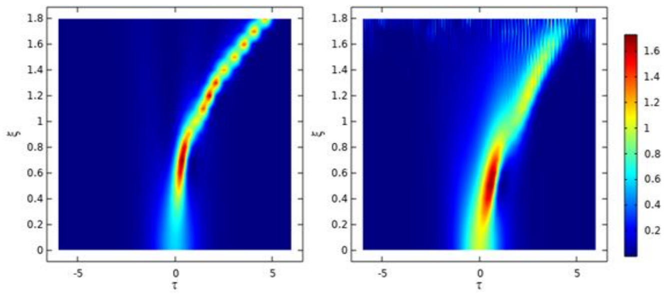

Figure 6: pulse evolution for *N*=2 soliton at $s = 0, \delta_3 = 0$ and $T_R = 0.1, 0.3$, respectively

Figures 9 to 12 represents the pulse shape at different propagation distances for cases *N*=1,2, respectively, where the left subfigure indicates $\delta_3 = 0$ and the right subfigure indicates $\delta_3 = 0.002$. **Figure** 9 represents case *N*=1 did not show significant changes in the shape of the pulse and periodicity can be kept. We did not notice a difference between the two cases $\delta_3 = 0, 0.002$.

Figure 10 represents the case *N*=2. Here, there are significant effects in the shape of the pulse, and there are no displacements or changes in the slope of the leading edge of the pulse. Note that there are very slight differences due to the change in δ_3.

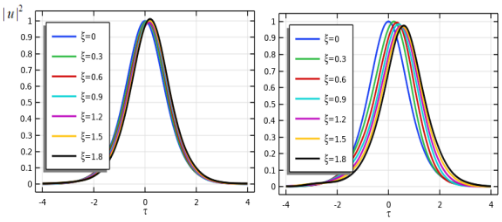

Figure 7: pulse evolution for N=1 soliton at $T_R = 0, \delta_3 = 0$ and $s = 0.1, 0.3$, respectively

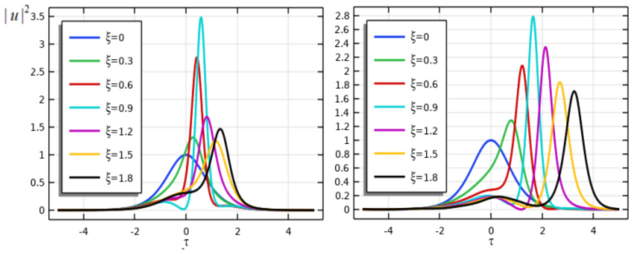

Figure 8: pulse evolution for N=2 soliton at $T_R = 0, \delta_3 = 0$ and $s = 0.1, 0.3$, respectively

Figure 11 represents the pulse shape for several distances in case *N*=1 at $\delta_3 = 0.002$, where the left subfigure indicates case $T_R = 0.1$ and the right subfigure indicates case $T_R = 0.3$. By comparing **Figure** 11 with 9 the effect of adding SRS is clear, as the center of the pulse will shift to the right with increasing distance and this displacement increase with the value of T_R. **Figure** 12 represents the pulse shape for several distances for case *N*=2 at $\delta_3 = 0.002$, where the left subfigure indicates case $T_R = 0.1$ and the right figure indicates case $T_R = 0.3$. Comparing **Figure** 12 with 10, we note that the pulse suffers from higher distortions and shifts proportional to the propagation distance by introducing the TOD effects.

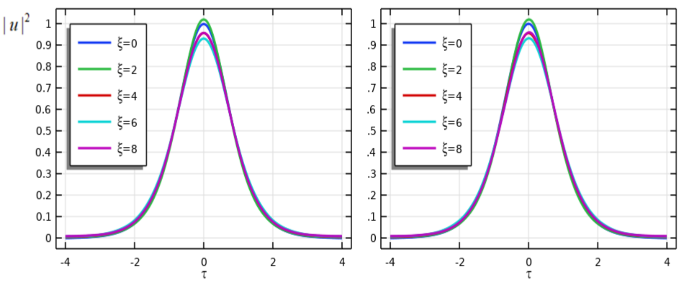

Figure 9: pulse evolution for N=1 soliton at $T_R = 0, s = 0$ and $\delta_3 = 0, 0.002$, respectively

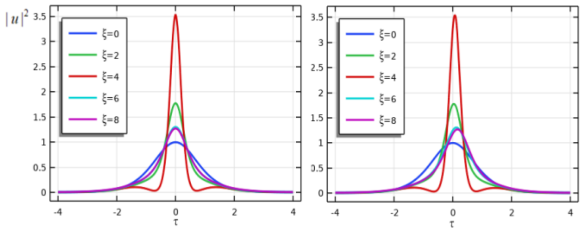

Figure 10: pulse evolution for N=2 soliton at $T_R = 0, s = 0$ and $\delta_3 = 0, 0.002$, respectively

Figures 13 to 21 represent the soliton pair propagation spectrum for the case *N*=1,2,3, respectively, at the conditions $C = 0, \phi = s = T_R = \delta_3 = 0, r = 1$, where the different subfigures indicate to the values $q = 2, 2.5, 3$. We notice from **Figure** 13 that the soliton pair gets closer together with the evolution in distance to from a single pulse of greater intensity. The distance required for the solitons union increases with increase of *q*, as it was $\xi = 4.5$ at *q*= 2, $\xi = 8$ at *q*=2.5 and $\xi = 13.5$ at *q*=3. Physically, the interaction with distance increases when the soliton pair appoaches each other, and then the solitons separate to achieve periodic behavior as long as *N*=1. **Figure** 14 represents the case *N*=2. We notice that the periodic behavior remains regular, but a decrease in *q* causes an increase in the interaction between the pair of solitons. The increase in interaction is accompanied by the generation of a secondary pulse between the pair of input pulses. **Figure** 15 represents the case *N*=3. It shows the interaction between the solitons with distance, and this interaction appears with less effect with increasing *q*. Searching for periodicals needs to work in larger propagation distances.

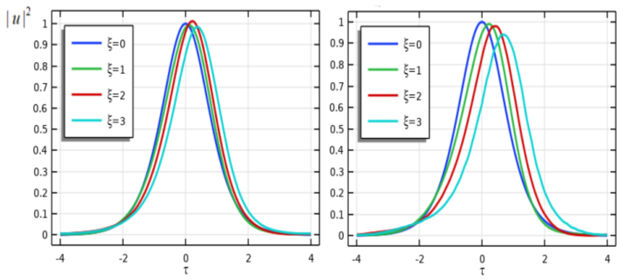

Figure 11: pulse evolution for *N*=1 soliton at $s = 0, \delta_3 = 0.002$ and $T_R = 0.1, 0.2$, respectively

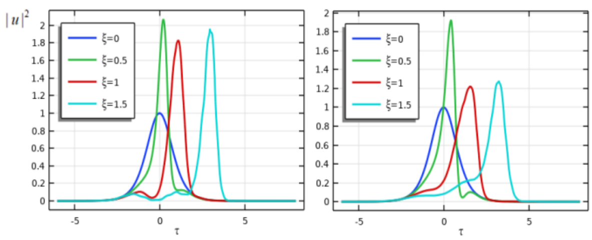

Figure 12: pulse evolution for *N*=2 soliton at $s = 0, \delta_3 = 0.002$ and $T_R = 0.1, 0.2$, respectively

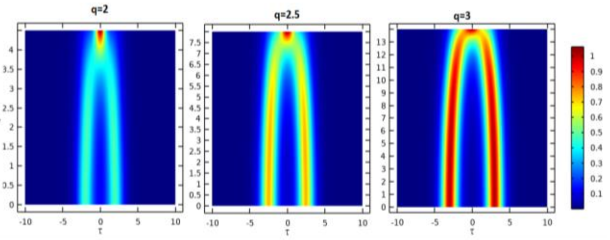

Figure 13: soliton interaction for N=1 at $\phi = s = T_R = \delta_3 = 0, r = 1$ for many values of q

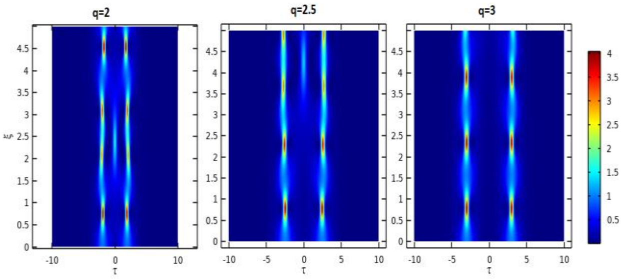

Figure 14: soliton interaction for N=2 at $\phi = s = T_R = \delta_3 = 0, r = 1$ for many values of q

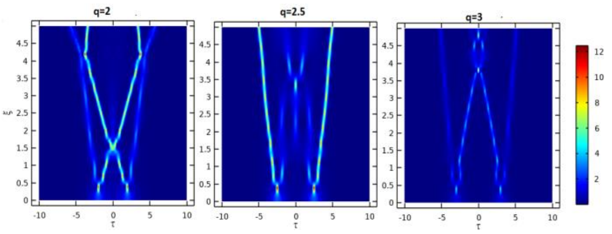

Figure 15: soliton interaction for N=3 at $\phi = s = T_R = \delta_3 = 0, r = 1$ for many values of q

Figure 16 represents the propagation spectrum of the pulse at N=1, q=2 and $\phi = 0$ without higher order nonlinear effects of the cases r= 1, 1.1, 1.2. It is clear from the figure that cases r=1, is the same as **Figure** 13 but cases r= 1.1, 1.2, will cause the power to be transferred to the pulse of greater intensity and the soliton pair union issue will disappear here compared to case r=1. **Figure** 17 it represents the propagation spectrum of the pulse at N=1, q=2 and r=1, in the absence of high order nonlinear effects for the cases ϕ=-π/4, 0, π/4. The case $\phi = 0$ similar to the previous figure, but if it is $\phi = \pi / 4$ then the delayed pulse will increase in intensity at the expense of the advanced pulse, while in the case of $\phi = -\pi / 4$, the opposite will happen. In both cases, the periodic behavior will not appear clearly.

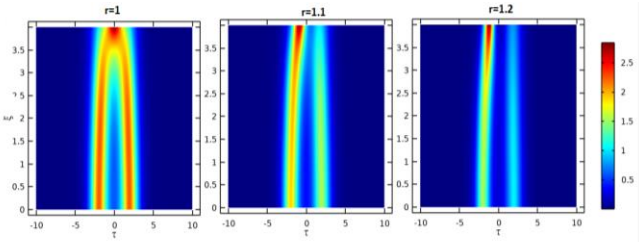

Figure 16: soliton interaction for N=1 at $\phi = s = T_R = \delta_3 = 0, q = 2$ for many values of r

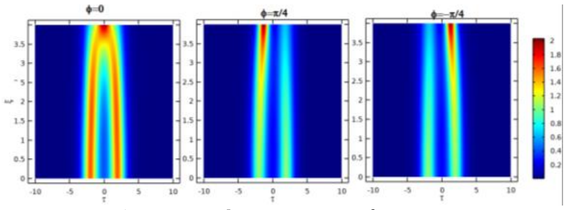

Figure 17: soliton interaction for N=1 at $s = T_R = \delta_3 = 0, q = 2, r = 1$ for many values of ϕ

Figure 18 obtains the soliton N=1, $T_R = \delta_3 = 0$, q=2, r=1 and ϕ=0 for the cases s=0, 0.1, 0.2, respectively. The cases s=0, 0.1, it is not different from the previous figure, but increasing s will cause the energy to transfer from one pulse to another with a rightward shift of the center of the interacting solitons. **Figure** 19 represents the case at N=2, $s = \delta_3 = 0$, q=2, r=1 and $\phi = 0$ for the cases $T_R = 0.1, 0.2, 0.3$, respectively. It is clear from the figure that the energy is transformed from the advanced pulse to the delayed pulse, and the amount of energy converted increases with the increase of T_R. As well as, the deviation of the center of the interacting solitons also increases with T_R.

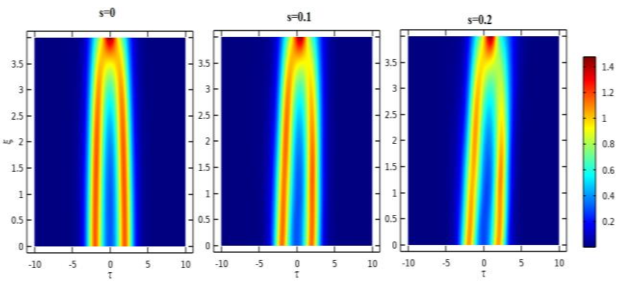

Figure 18: soliton interaction for N=1 at $T_R = \delta_3 = 0, q = 2, r = 1, \phi = 0$ for many values of s

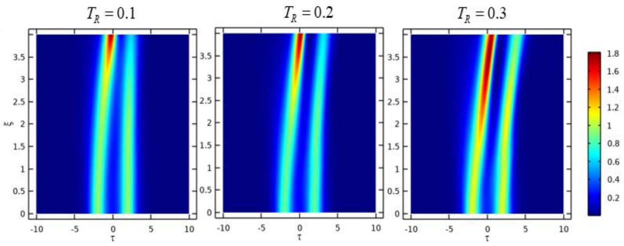

Figure 19: soliton interaction for N=1 at

$s=\delta_3=0, q=2, r=1, \phi=0$ for many values of T_R

Figure 20 shows the soliton *N*=3, $T_R = s = 0$, *q*=2, *r*=1 and ϕ=0 for cases $\delta_3 = 0, 0.001, 0.002$, respectively. At $\delta_3 = 0$ the pulse pair will interact to merge through a certain distance and then separate with and presence of two weak pulses out of context of the soliton pair. When $\delta_3 \neq 0$, the merging and splitting mechanism will not fully occur and the periodic behavior will be lost.

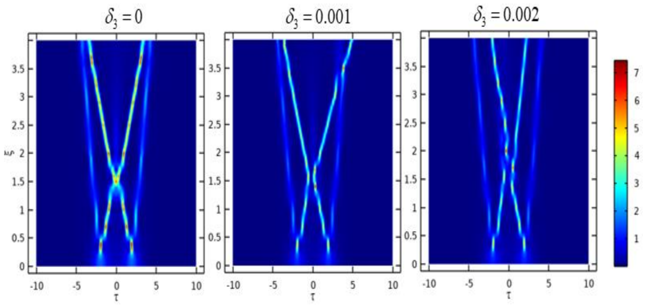

Figure 20: soliton interaction for N=3 at

$s=T_R=0, q=2, r=1, \phi=0$ for many values of δ_3

VI. Conclusions

The presence of third-order dispersion, SRS, and self-steepening significantly affect the properties of both fundamental solitons and higher-order solitons. These effects can cause temporal broadening or compression, spectral broadening due to energy transfer, and pulse reshaping. The fundamental soliton stands out for its ability to maintain its shape without undergoing any changes during propagation. The stability of the fundamental soliton is a result of the delicate balance between the nonlinear and dispersive effects, which are carefully engineered in optical fiber systems. However, it is important to note that in certain scenarios, solitons may exhibit periodic behavior with variations in their shape and characteristics. The periodicity of solitons depends on specific conditions such as the magnitude of nonlinearity, dispersion coefficients, and initial conditions. Understanding and controlling these phenomena are crucial for applications in nonlinear optics and fiber optics communication systems. These variations can be observed in higher-order solitons or solitons propagating in dispersive media, where the interplay between nonlinearity and dispersion leads to more complex dynamics.

Conflict of Interest

Authors declare that they have no conflict of interest.

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