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Photoexcitation of ^{13}C, ^{15}N, ^{17}O, ^{17}F and ^{39}K in the Framework of Particle-Core Coupling Model (PCM)

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Abstract

Photoexcitation of the collective low-lying states in ^{13}C, ^{15}N, ^{17}O, ^{17}F and ^{39}K are studied in the framework of Particle-Core Coupling Model (**PCM**). According to PCM, the total Hamiltonian is diagonalized in the configuration mixing space of the basis states which consist of the core states that restricted to at most three quadrupole and two octupole phonon excitations and harmonic oscillator single-particle states. The resulting eigenvalues, correspond to the energy levels and the eigenvectors, are used to calculate the magnetic dipole moment μ(n.m.), electric quadrupole moment Q($e\cdot fm^2$), and reduced transition probability B(M1)↓$(n.m.)^2$ and B(E2)↓$(e^2\cdot fm^4)$. The core polarization effects are introduced by giving the odd nucleon an effective charge and effective g-factor to obtain the best description for data.

Keywords: Particle-Core Coupling Model (**PCM**), magnetic dipole moment μ, electric quadrupole moment Q, reduced magnetic and electric transition probabilities B(M1)↓, B(E2)↓, respectively.

1. Introduction

The study of odd-mass nuclei near closed shells has long been a subject of interest due to the simple structure of their excitation spectra. Especially, nuclei with a few particles (or holes) outside (or inside) a closed shell have low-lying excitation spectra that are largely characterized as transitions of a single particle between shell-model orbitals. This simple picture is often far from the realistic one. The core degrees of freedom must be taken into account [1].

The PCM is such a model where an odd-particle (hole or quasi-particle) moving in the field of a vibrating, collective nucleus becomes coupled to these collective vibrations in a very natural way and is useful in particular in the rare-earth region (A≈150) and in the neighborhood of ^{208}Pb, where the doubly even nuclei show the characteristics of vibrational spectra [2]. The description of the vibrations of the core in terms of this model is based upon an analogy with the classical vibration of a liquid drop [3].

The aim of PCM study presented in this work was two field: (i) Investigation if ^{12}C, ^{16}O and ^{40}Ca are good cores for their adjacent odd nuclei. (ii) Extracting information about the ability of the PCM to describe the structure of ^{13}C, ^{15}N, ^{17}O, ^{17}F and ^{39}K.

2. Theory

2.1 The Hamiltonian of PCM

In the PCM, the configuration mixing by a particle-core coupling Hamiltonian is fully taken into account. The configuration space is a direct product of the single particle (s.p) states $\left| jm_j \right\rangle$ for the last odd particle and the vibrational states of the core [4]:

$$\left|\left(N_3R_3N_2R_2\right)R, j; IM\right\rangle = \sum_{M_{R3},M_{R2}} \sum_{M_R,m_j} \left\langle R_3M_{R_3}R_2M_{R_2} \middle| RM_R\right\rangle\left\langle RM_R\, jm \middle| IM\right\rangle \left| N_3R_3M_{R_3}\right\rangle\left| N_2R_2M_{R_2}\right\rangle\left| jm_j\right\rangle \quad (1)$$

where the collective wave function with total angular momentum $\left|R\right\rangle$ is coupled with the s.p state $\left| jm_j \right\rangle$ to give a total angular momentum $\left|IM\right\rangle$ and projection M along the z-axis, while the collective total angular momentum $\left|R\right\rangle$ is formed by coupling of an N_3=0,1 and 2 octupole phonon state with angular momentum $\left|R_3\right\rangle$ with an N_2=0,1,2 and 3 quadrupole phonon state with angular momentum $\left|R_2\right\rangle$. The total PCM Hamiltonian is [5]:

$$H = H_{coll} + H_{s.p} + H_{\text{int}} \quad (2)$$

Using the standard quantization procedure, one can introduce creation and annihilation operators for phonon b^+_{LM}, b_{LM} and for particle a^+_{jm}, a_{jm} to write the collective vibration Hamiltonian of the core H_{coll},

$$H_{coll} = \sum_{LM} \hbar\omega_L [b^{+}_{LM} b_{LM} + \tfrac{1}{2}] \quad , \qquad (3)$$

where the quantum of energy, $\hbar\omega_L$ (phonon energy), associated with the core vibrations, can be deduced from the neighboring even-even nucleus, which forms the core of odd-A nucleus, and the s.p Hamiltonian $H_{s.p}$,

$$H_{s.p} = \sum_{jm} E_j a^{+}_{jm} a_{jm} \quad , \qquad (4)$$

where the s.p energies E_j, can be deduced from experimental information. The odd nucleon interacts with the core vibrations, and the interaction Hamiltonian H_{int} [6],

$$H_{\text{int}} = -\sum_{LM} \left(\frac{\pi}{2L+1}\right)^{1/2} \xi_L \hbar\omega_L [b_{LM} + (-1)^M b^{+}_{L-M}] Y_{LM}(\hat{r}) \ . \qquad (5)$$

where the strength parameter ξ_L, for the coupling of the single nucleon, moves in the harmonic oscillator (HO) potential, with the vibrating core can be estimated to be [6]:

$$\xi_L = 41A^{-1/3}\left(N + \frac{3}{2}\right)\sqrt{\frac{2L+1}{2\pi\hbar\omega_L C_L}} \qquad (6)$$

where N is the number of harmonic oscillator quanta, which given by $N = 2(n-1)+l$, with n and l are the principal and orbital quantum numbers, and the stiffness parameter C_L refers to the rigidity of the core, which can be estimated from the experimental $B(EL\downarrow)$ values as [7]:

$$C_L = \left(\frac{3}{4\pi} ZeR_0^L\right)^2 \frac{\hbar\omega_L}{2B(EL\downarrow)} \qquad (7)$$

By substituting of eq.(7) in eq.(6), one obtains the strength parameter ξ_L for any multipolarity L. A and Z are the mass and atomic numbers of the core nucleus of equilibrium radius $R_0 = 1.2A^{1/3}$.

Diagonalization of the total Hamiltonian matrix elements between the mixed configurations of PCM states eq.(1), yields eigenvalues and eigenvectors, which are used to calculate the magnetic dipole moments μ, electric quadrupole moments Q, reduced magnetic and electric transition probabilities B(M1)↓, B(E2)↓ from the first excited state to the ground state, respectively, as follow:

2.2 Transition probabilities and Moments

2.2.1 Magnetic Dipole Moment

The expression of the magnetic dipole moment in the framework of PCM is given by [7]:

$$\mu = \frac{I}{(I+1)(2I+1)} \sum_{RR',jj'} C_\alpha(R',j';I) C_\alpha(R,j;I) \Bigg[(-1)^{R+I+j} (2I+1) \hat{j}\hat{j}' \begin{Bmatrix} j' & j & 1 \\ I & I & R' \end{Bmatrix} \delta_{RR'}$$
$$\times \left\{ g_s \sqrt{\frac{3}{2}} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ j & j' & l' \end{Bmatrix} \delta_{ll'} + g_l \hat{l} \sqrt{l(l+1)} \begin{Bmatrix} l' & l & 1 \\ j & j' & \frac{1}{2} \end{Bmatrix} \delta_{SS'} \right\} + (-1)^{R+I+j'} g_R \hat{R}\hat{R}' \delta_{jj'}$$
$$\times \left\{ \hat{R}_2 \sqrt{R_2(R_2+1)} \begin{Bmatrix} R' & R & 2 \\ I & I & j \end{Bmatrix} \begin{Bmatrix} R'_2 & R_2 & 2 \\ R & R' & R'_3 \end{Bmatrix} \delta_{N_3 N'_3} \delta_{R_3 R'_3} + \hat{R}_3 \sqrt{R_3(R_3+1)} \right. \tag{8}$$
$$\left. \times \begin{Bmatrix} R' & R & 3 \\ I & I & j \end{Bmatrix} \begin{Bmatrix} R'_3 & R_3 & 3 \\ R & R' & R_2 \end{Bmatrix} \delta_{N_2 N'_2} \delta_{R_2 R'_2} \right\} \Bigg] \mu_N$$

where $\mu_N = \dfrac{e\hbar}{2m_p c} = 0.1051 e \cdot fm = 1 n.m.$, with m_p is the proton mass. g_l and g_s are the orbital and spin nucleon g-factors, respectively, where $g_l(p) = 1$, $g_l(n) = 0$, $g_s(p) = 5.586$, $g_s(n) = -3.826$, and the g-factor for the core is represented by $g_R = Z/A$, where $\hat{j} = \sqrt{2j+1}$, $\hat{l} = \sqrt{2l+1}$.

2.2.2 <u>Reduced magnetic transition probability B(M1)↓</u>

The reduced magnetic transition probability B(M1)↓ in the case of one nucleon outside the core can be written as[7]:

$$B(\text{M1}; \alpha I \to \beta I') = \frac{1}{2I+1} \Bigg| \sqrt{\frac{3}{4\pi}} \sum_{RR'jj'} C_\beta(R',j';I') C_\alpha(R,j;I) \Bigg[(-1)^{R+I'+j'} \hat{I}\hat{I}\hat{j}\hat{j}' \begin{Bmatrix} j' & j & 1 \\ I & I' & R' \end{Bmatrix} \delta_{RR'}$$
$$\times \left\{ g_s \sqrt{\frac{3}{2}} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ j & j' & l' \end{Bmatrix} \delta_{ll'} + g_l \hat{l} \sqrt{l(l+1)} \begin{Bmatrix} l' & l & 1 \\ j & j' & \frac{1}{2} \end{Bmatrix} \delta_{SS'} \right\} + (-1)^{R+I+j'} g_R \hat{R}\hat{R}' \delta_{jj'}$$
$$\times \left\{ \hat{R}_2 \sqrt{R_2(R_2+1)} \begin{Bmatrix} R' & R & 2 \\ I & I' & j \end{Bmatrix} \begin{Bmatrix} R'_2 & R_2 & 2 \\ R & R' & R'_3 \end{Bmatrix} \delta_{N_3 N''_3} \delta_{R_3 R'_3} + \hat{R}_3 \sqrt{R_3(R_3+1)} \right. \tag{9}$$
$$\left. \times \begin{Bmatrix} R' & R & 3 \\ I & I' & j \end{Bmatrix} \begin{Bmatrix} R'_3 & R_3 & 3 \\ R & R' & R_2 \end{Bmatrix} \delta_{N_2 N'_2} \delta_{R_2 R'_2} \right\} \Bigg] \mu_N \Bigg|^2$$

2.2.3 <u>Electric-Quadrupole Moment</u>

The expression of the electric quadrupole moment Q in the framework of PCM is given by [7]:

$$Q=\sqrt{\frac{16\pi}{5}}\sqrt{\frac{I(2I+1)}{(I+1)(2I+1)(2I+3)}}\sum_{ll'jj'N_2R_2N_3R_3RR'}C_\alpha(R',j';I)C_\alpha(R,j;I)$$

$$\times(-1)^{I+j'}(2I+1)\left\{\left(e+\frac{Ze}{A^2}\right)(-1)^{R+j}\,\hat{j}\hat{j}'\sqrt{\frac{5}{4\pi}}\begin{pmatrix} j' & 2 & j \\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\begin{Bmatrix} j' & j & 2 \\ I & I & R'\end{Bmatrix}\delta_{even}^{l'+l}\langle n'l'|r^2|nl\rangle\delta_{RR'}\right.$$

$$+\frac{3}{4\pi}ZeR_0^2\sqrt{\frac{\hbar\omega_2}{2C_2}}\left\{(-1)^{R'}\hat{R}\hat{R}'\begin{Bmatrix} R' & R & 2 \\ I & I & j\end{Bmatrix}\begin{Bmatrix} R_2 & R_2' & 2 \\ R' & R & R_3\end{Bmatrix}\delta_{jj'}\delta_{N_3N_3'}\delta_{R_3R_3'}\right.$$

$$\left.\left.\times\left[(-1)^{R'}\langle N_2'R_2'\|b_2^+\|N_2R_2\rangle+(-1)^{R}\langle N_2R_2\|b_2^+\|N_2'R_2'\rangle\right]\right\}\right\} \qquad (10)$$

where the radial integrals of r^2 between the harmonic oscillator s.p wave functions are denoted by $\langle n'l'|r^2|nl\rangle$, and $\delta_{even}^{l'+l}=\frac{1}{2}\{1+(-1)^{l'+l}\}$, with l and l' are the orbital quantum number for the initial and final state of the odd nucleon, respectively. The reduced matrix elements of the quadrupole and octupole boson creation operators can be regarded as boson coefficient of fractional parentage (**BCFP**) given in ref.[8].

2.2.4 Reduced electric transition probability B(E2)↓

The reduced electric transition probability B(E2)↓ is given in the framework of PCM by [7]:

$$B(E2;\alpha I\to\beta I')=\frac{1}{2I+1}\left|\sum_{ll'jj'N_2R_2N_3R_3RR'}C_\beta(R',j';I')C_\alpha(R,j;I)\right.$$

$$\times(-1)^{I+j'}\hat{I}\hat{I}'\left\{\left(e+\frac{Ze}{A^2}\right)(-1)^{R+j}\,\hat{j}\hat{j}'\sqrt{\frac{5}{4\pi}}\begin{pmatrix} j' & 2 & j \\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\begin{Bmatrix} j' & j & 2 \\ I & I' & R'\end{Bmatrix}\delta_{even}^{l'+l}\langle n'l'|r^2|nl\rangle\delta_{RR'}\right.$$

$$+\frac{3}{4\pi}ZeR_0^2\sqrt{\frac{\hbar\omega_2}{2C_2}}\left\{(-1)^{R'}\hat{R}\hat{R}'\begin{Bmatrix} R' & R & 2 \\ I & I' & j\end{Bmatrix}\begin{Bmatrix} R_2 & R_2' & 2 \\ R' & R & R_3\end{Bmatrix}\delta_{jj'}\delta_{N_3N_3'}\delta_{R_3R_3'}\right.$$

$$\left.\left.\left.\times\left[(-1)^{R'}\langle N_2'R_2'\|b_2^+\|N_2R_2\rangle+(-1)^{R}\langle N_2R_2\|b_2^+\|N_2'R_2'\rangle\right]\right\}\right\}\right|^2 \qquad (11)$$

where the coefficients C_α and C_β that appear in eqs. (8) - (11) are the eigenvectors for the initial and final states, respectively, which are obtained from the diagonalization of eq.(2).

3. Results and discussion

According to the PCM, ^{13}C and ^{17}O nuclei are represented by a neutron particle ν outside the closed shell nuclei ^{12}C and ^{16}O, respectively

$${}^{17}_{8}O_9={}^{16}_{8}O+\nu(1d_{5/2})$$

and the nucleus ^{17}F is represented by a proton particle π outside the closed shell nucleus^{16}O

$$^{17}_{9}F_{8} = ^{16}_{8}O + \pi(1d_{5/2})$$

while the nuclei ^{15}N and ^{39}K are represented by a proton hole inside the closed shell nuclei ^{16}O and ^{40}Ca, respectively

$$^{15}_{7}N_{8} = ^{16}_{8}O + h\,(1p^{-1}_{1/2})$$

$$^{39}_{19}K = ^{40}_{20}Ca + h(1d^{-1}_{3/2})$$

The total Hamiltonian which is given in eq.(2) is diagonalized in the configuration mixing space of the basis state, as in eq(1). An estimation of the parameters E_j , $\hbar\omega_L$ and ξ_L is required. The unperturbed energy E_j of the odd nucleon states that taken into account are measured experimentally [9, 10, 11] and given in table (1).

Table (1): The unperturbed energies (in MeV) for the s.p states that taken into account.

Nucleus	**s.p state**	E_j **(MeV)**
$^{13}_{6}C$ $^{(a)}$	$1p_{1/2}$ $2s_{1/2}$ $1d_{5/2}$ $1d_{3/2}$	0.00 3.09 3.85 8.34
$^{15}_{7}N$ $^{(b)}$	$1p^{-1}_{1/2}$ $1p^{-1}_{3/2}$ $1s^{-1}_{1/2}$	0.00 6.32 27.9
$^{17}_{8}O$ $^{(c)}$	$1d_{5/2}$ $2s_{1/2}$ $1d_{3/2}$	0.00 0.87 5.08
$^{17}_{9}F$ $^{(b)}$	$1d_{5/2}$ $2s_{1/2}$ $1d_{3/2}$	0.00 0.495 5.10
$^{39}_{19}K$ $^{(b)}$	$1d^{-1}_{3/2}$ $2s^{-1}_{1/2}$ $1d^{-1}_{5/2}$	0.00 2.52 5.01

a)Ref.[11] b)Ref.[9] c)Ref.[10]

The quadrupole and octupole phonon energies are taken from experimental excitation energies [12, 13, 14] for the transitions ($0^+_1 \rightarrow 2^+_1$) and ($0^+_1 \rightarrow 3^-_1$) in the core nucleus, and they are given with the corresponding reduced transition rates B(EL)↓ in table (2).

Table (2): The core nucleus parameters (^{12}C, ^{16}O and ^{40}Ca).

Core nucleus	$\hbar\omega_2$ (MeV)	$\hbar\omega_3$ (MeV)	**B(E2)↓** $e^2\,fm^4$	**B(E3)↓** $e^2\,fm^6$
^{12}**C** $^{(a)}$	4.439	9.641	7.996	107.0
^{16}**O** $^{(b)}$	6.917	6.13	4.6 ± 1.0 $^{(c)}$	212.1 $^{(d)}$
^{40}**Ca** $^{(d)}$	3.900	3.730	16.8	3014.3

a) Ref [12] b) Ref [13] c) Ref [14] d) Ref [11].

The strength parameters of the interaction of the s.p with the quadrupole and octupole phonons ξ_2 and ξ_3 are calculated according to eq.(6), and the surface stiffness parameter C_2 and C_3 are calculated according to eq.(7). All of these results are given in table (3).

Table (3): The coupling strength ξ_L , stiffness parameter C_L , and gyromagnetic ratio of the core g_R .

parameters	$^{13}_{6}C$	$^{15}_{7}N$	$^{17}_{8}O$	$^{17}_{9}F$	$^{39}_{19}K$
ξ_2	3.241	0.931	1.250	1.250	0.698
ξ_3	2.351	2.791	3.748	3.748	2.818
C_2	32	229	229	229	750
C_3	40	40	40	40	67
$g_R = Z/A$	0.461	0.466	0.470	0.529	0.487

3.1 μ $(n.m.)$ and B(M1)↓ $(n.m.)^2$ calculations

Besides the vibrational motion of the core, another effect must be taken into account to reproduce the measured quantities. This additional effect is the core polarization which carried out by giving the spin gyromagnetic factor of the odd nucleon (or hole) an effective value $g_s^{eff}(p/n)$ less than that of its free value $g_s^{free}(p/n)$.

The eigenvectors of the initial and final states C_α and C_β are used to calculate the magnetic dipole moments μ given by eq.(8) and the reduced magnetic transition rates B(M1)↓ according to eq.(9). Our results are displayed in tables (4) and (5) for μ $(n.m.)$ and B(M1)↓ $(n.m.)^2$, respectively.

Table (4) displays the theoretical calculations [3] for magnetic ground state dipole moments which based on s.p model (fourth column) and PCM (fifth and sixth columns). An improvement of the PCM calculations is observed by introducing the core-polarization effects (effective g-factors) which given in the seventh column of this table. Thus, by a slight reduction of g_s^n-factors from their free values, the calculated magnetic dipole moment of ^{13}C and ^{17}O are reduced from 25.3 and 105.44 n.m. to 0.881 and -1.7 n.m., respectively, which are in reasonable agreement with the measured values that given in the third column of table (4).

The PCM calculation for the μ of the ground state of ^{15}N is in agreement with that of s.p prediction and both are slightly less than the experimental value. This result reflects the negligible contribution of the oscillating core to this property of the ground state of ^{15}N.

On the other side, the calculated magnetic dipole moment of the ground state of ^{17}F and ^{39}K using PCM and taking into account the effective g-factors are in much better agreement with the experimental values than that obtained from the extreme s.p (Schmidt) prediction.

Table (4): Magnetic ground state dipole moments $\mu(n.m.)$.

μ (n.m.)							
Nucleus	J^π	T	**Exp.**	**s.p model [3]**	**PCM**		$g_s^{eff}(p/n)$
					$g_s^{free}(p/n)$	$g_s^{eff}(p/n)$	
$^{13}_{6}C$	1/2⁻	1/2	0.7024 $^{(a)}$	0.64	25.3	0.881	$0.99\, g_s^n(free)$
$^{15}_{7}N$	1/2⁻	1/2	-0.283 $^{(b,c)}$	-0.264	-0.265		$g_s^h(free)$
$^{17}_{8}O$	5/2⁺	1/2	-1.894 $^{(c)}$	-1.913	105.44	-1.7	$0.998\, g_s^n(free)$
$^{17}_{9}F$	5/2⁺	1/2	4.722 $^{(c)}$	4.793	4.80	4.72	$0.97\, g_s^P(free)$
$^{39}_{19}K$	3/2⁺	1/2	0.3914 $^{(d)}$	0.124	0.132	0.393	$0.843\, g_s^P(free)$

a)Ref.[15] b) Ref.[11] c) Ref.[4] d)Ref.[16]

Table (5): Reduced magnetic transition rates B(M1)↓ $(n.m.)^2$.

B(M1)↓ $(n.m.)^2$						
Nucleus	$J_i^\pi \to J_f^\pi$	E_x (MeV)	**Exp.**	**PCM**		$g_s^{eff}(p/n)$
				$g_s^{free}(p/n)$	$g_s^{eff}(p/n)$	
$^{13}_{6}C$	$3/2^- \to 1/2^-$	3.68	0.617 $^{(a)}$	5.52	0.578	$0.99\, g_s^n(free)$
$^{15}_{7}N$	$3/2^- \to 1/2^-$	6.32	1.16±0.24 $^{(b)}$	0.194		$g_s^h(free)$
$^{39}_{19}K$	$1/2^+ \to 3/2^+$	2.53	0.030 $^{(c)}$	0.0552	0.031	$0.79\, g_s^P(free)$

a) Ref. [17] b) Ref.[18] c) Ref. [14], where $g_s^P(free) = g_s^h(free)$ =5.586, $g_s^n(free)$ = -3.826, $g_l(eff) \approx g_l(free)$.

3.2 $Q(e \cdot fm^2)$ and B(E2)↓ $(e^2.fm^4)$ calculation

In our work, the electric quadrupole moment $Q(e \cdot fm^2)$ of the ground state, and the reduced electric transition rates B(E2)↓ from the first excited states to the ground state are calculated for ^{13}C, ^{15}N, ^{17}O, ^{17}F and ^{39}K by using PCM according to eqs.(10) and (11), respectively.

Core polarization effects are taken into account by giving the odd nucleon (or hole) an effective charge $\tilde{e}(p/n)$ different from that given to the free one $e(p/n)$. The results are displayed in tables (6) and (7) for $Q(e \cdot fm^2)$ and B(E2)↓$(e^2.fm^4)$, respectively.

Table (6) displays the calculations for electric ground state quadrupole moment $Q(e \cdot fm^2)$ which based on s.p model with free and effective charge of the odd nucleon (fourth and fifth columns, respectively) and **PCM** with free and effective charge of the odd nucleon (seventh and eight columns, respectively). The effective charges of the odd nucleon are given in the sixth and ninth columns, for the s.p and PCM models, respectively.

Table (6): Electric ground state quadrupole moment $Q(e \cdot fm^2)$.

Nucleus	J^π T	$Q_{exp.}$	s.p model			PCM		
			$Q(e)$	$Q(\tilde{e})$	$\tilde{e}$	$Q(e)$	$Q(\tilde{e})$	$\tilde{e}$
$^{17}_{8}O$	$5/2^+$ 1/2	-2.58 $^{(a)}$	0.0	-2.63	0.4e	-2.4	-2.6	0.04e
$^{17}_{9}F$	$5/2^+$ 1/2	±10.0 $^{(b)}$	-7.657	-10.1	1.32e	-7.564	-10.0	1.49e
$^{39}_{19}K$	$3/2^+$ 1/2	5.4±0.2 $^{(c)}$	5.32 $^{(d)}$	5.46	1.05e	-6.20		e

a) Ref.[19], b)Ref.[20, 21], c)Ref.[22, 23], d) Ref.[24].

The calculated $Q(e \cdot fm^2)$ of the ground state of $^{17}_{8}O$ using PCM and taking into account the effective charge is in much better agreement with the experimental values than that obtained from the s.p model with effective charge. However, the microscopic core polarization effects calculation for the electric ground state quadrupole moment of $^{17}_{8}O$ was found to be **(-2.84** $e \cdot fm^2$**)** [23], which are slightly shifted from the experimental value.

A very well reproduction of the experimental value of the electric quadrupole moment of the ground state of $^{17}_{9}F$ is obtained by giving the odd-proton an effective charge equal to **1.49e** in the framework of PCM.

The Q of ${}^{39}_{19}K$ has the experimental value **5.4±0.2** $e\cdot fm^2$ [22]. The calculated Q of the ground state of ${}^{39}_{19}K$ using s.p. model is in good agreement with experimental value. A calculation with microscopic core polarization effects predicts this value of Q at **7.12** $e\cdot fm^2$ [23]. While our PCM calculation predicts a negative value.

For a single-proton hole, the quadrupole moment is $-Q_{s.p}$. The opposite sign of the quadrupole moments for a particle and a hole correspond to the fact that the quadrupole operator, being a function of the position coordinates, transforms under particle-hole conjugation with the phase (-1)[9].

The calculations of the B(E2)↓ for the transition from $3/2^-$ 1/2 (3.68MeV), $3/2^-$ 1/2 (6.32MeV), $1/2^+$ 1/2 (0.87 MeV), $1/2^+$ 1/2 (0.50 MeV) and $1/2^+$ 1/2 (2.536 MeV) to the ground state in ^{13}C, ^{15}N, ^{17}O, ^{17}F and ^{39}K, respectively, are shown in table (7).

The calculated value of B(E2)↓ using s.p model for the transition ($3/2^- \rightarrow 1/2^-$) in ^{15}N is found to be **4.6** $e^2\cdot fm^4$ in comparison with the measured value **7.4±0.25** $e^2\cdot fm^4$ [11]. According to the PCM, a very well agreement with the measured value is obtained when we use effective charge $\tilde{e}(h) =$ **1.1e** for the valence proton-hole for ^{15}N.

The calculated B(E2)↓ value using s.p model for the transition ($1/2^+ \rightarrow 5/2^+$) in ^{17}O is zero due to the neutral charge of the odd neutron. The PCM result using the value of free neutron is less than the experimental data by a factor of **~2**. To reproduce the measured B(E2)↓ value for this transition in ^{17}O according to PCM, we give the odd neutron an effective charge equal to **0.18e** as a result of the core polarization effects.

Table (7): Reduced electric transition rates B(E2)↓ $(e^2\cdot fm^4)$.

Nucleus	$B(E2)\downarrow_{exp.}$	s.p model			PCM		
		$B(E2)_e$	$B(E2)_{\tilde{e}}$	$\tilde{e}$	$B(E2)_e$	$B(E2)_{\tilde{e}}$	$\tilde{e}$
$^{13}C\,(\frac{3}{2}^- \rightarrow \frac{1}{2}^-)$ $E_\gamma = 3.68$ MeV	6.46±0.275 $^{(a)}$	0.0	2.86	e	8.843 6.30		C_2 =32 C_2 =45
$^{15}N\,(\frac{3}{2}^- \rightarrow \frac{1}{2}^-)$ $E_\gamma = 6.32$ MeV	7.4±0.25 $^{(b)}$	4.6	7.774	1.3e	6.77	7.47	1.1e
$^{17}O\,(\frac{1}{2}^+ \rightarrow \frac{5}{2}^+)$ $E_\gamma = 0.87$ MeV	6.54±0.48 $^{(b)}$	0.0	6.36	0.43e	3.783	6.52	0.18e
$^{17}F\,(\frac{1}{2}^+ \rightarrow \frac{5}{2}^+)$ $E_\gamma = 0.5$ MeV	64.0 $^{(b)}$	43	64.0	1.22e	35.66	64.33	1.55e
$^{39}K\,(\frac{1}{2}^+ \rightarrow \frac{5}{2}^+)$ $E_\gamma = 2.52$ MeV	37.8±3.6 $^{(c)}$	27	38.88	1.2e	2.8	30.3	C_2 =750 C_2 =1500

a) Ref.[2] b) Ref.[9] c) Ref.[20] , where $\tilde{e}(h) = \tilde{e}(p)$ and C_2 is the surface stiffness parameter, $\tilde{e}(h) = 2.0$e for ${}^{39}_{19}K$.

The s.p model for ^{17}F represents a proton outside a closed, negligible 1p-shell core. For the transition $(1/2^+ \rightarrow 5/2^+)$ the s.p model result underestimates the measured value of the transition strength by a factor of ~**1.5**. The inclusion of the core polarization effects through effective charges with the PCM reproduces the measured B(E2)↓ value correctly, as given in the sixth row of table(7).

The s.p model predicts the values **0.0** and $\mathbf{27}\, e^2.fm^4$ for the electric quadrupole strengths of transitions $(3/2^- \rightarrow 1/2^-)$ and $(1/2^+ \rightarrow 3/2^+)$ in ^{13}C and ^{39}K, respectively.

Our PCM calculations for ^{13}C using the value of free neutron charge over-predicts the measured B(E2)↓ value by a factor of ~1.37. A very well reproduction of this value is obtained by increasing the calculated stiffness parameter C_2 of the core nucleus by a factor of **~1.4**. While the coupling of the oscillating core with the odd proton-hole of bare charge in ^{39}K found the strength of the transition $(1/2^+ \rightarrow 3/2^+)$ weaker than that predicted by s.p model. However, giving the proton-hole an effective charge equal to **2e** and duplicating the calculated C_2 value reproduces the measured value of B(E2)↓.

Conclusions

In this work, the structures of the open-shell nuclei ^{13}C, ^{15}N, ^{17}O, ^{17}F and ^{39}K were studied in the framework of collective model PCM. From the comparison of the present calculations with the experimental data we can draw the following conclusions:

1. The PCM with first order in the collective variables α_{LM} is succeeded in describing the electromagnetic moments, transition rates B(M1)↓ and B(E2)↓ for open-shell nuclei in *p*- and *sd*-shell regions.
2. In our PCM based calculation of the electromagnetic moments and transition rates, the effective charges are introduced and found to be of the order: $\tilde{e}(p)$ =1.49e~2.0e and $\tilde{e}(n)$ = 0.04e~0.18e .
3. We found that ^{12}C, ^{16}O, and ^{40}Ca are good cores for the adjacent odd nuclei.

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الأستثارة الفوتونية للنوى $^{13}C, ^{15}N, ^{17}O, ^{17}F$ و ^{39}K في أطار نموذج اقتران جسيم- لبُ

Particle-Core Coupling Model (PCM)

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الخلاصة

تمت دراسة الاستثارة الفوتونية لانتقالات المستويات التجميعية للنوى $^{13}C, ^{15}N, ^{17}O, ^{17}F$ و ^{39}K في أطار نموذج اقتران جسيم- لبُ **Particle-Core Coupling Model (PCM)** .

طبقا لنموذج **PCM** ، تم تحويل المؤثر الهاملتوني الكلي إلى مصفوفة قطرية ضمن فضاء التشكيلات الممتزجة للحالة الأساسية التي تتألف من حالة اللبُ المحددة بتهيجات ثلاث فونونات رباعية القطب وفونونين ثمانية القطب على الأغلب وحالات الجسيم-المفرد للمتذبذب التوافقي.

تم استخدام القيم الذاتية والدوال الذاتية الناتجة في حساب عزوم ثنائيات القطب المغناطيسية **μ(n.m.)** وعزوم رباعيات القطب الكهربائية $Q\ (e\cdot fm^2)$ واحتمالية الانتقال المغناطيسية المختزلة $B(M1)\downarrow (n.m.)^2$ واحتمالية الانتقال الكهربائية المختزلة $B(E2)\downarrow (e^2\cdot fm^4)$ **0** أدخلُ تأثير استقطاب اللبُ باستخدام الشحنة الفعالة وعوامل-**g** الفعالة للحصول على نتائج جيدة ومتوافقة مع النتائج العملية.