University of Thi-Qar Journal Of Science (UTsci)
Website: http://jsci.utq.edu.iq Email: utjsci@utq.edu.iq
Volume 6, Number 3, December 2017

On Intuitionistic Fuzzy Minimal Semi-preopen Set

Afakar Kareem Mnahi

Ministry of Education- Department of Education -Thi Qar University

E-mail: afkarkareem@yahoo.com

Abstract:

In this paper, we introduce the concept of intuitionistic fuzzy minimal open sets on intuitionistic fuzzy topological space. We introduce the concepts of closure and interior defined by intuitionistic fuzzy minimal open set and intuitionistic fuzzy minimal closed set. We also introduce a intuitionistic fuzzy minimal semi-preopen and intuitionistic fuzzy minimal semi-preclosed. We also redefine the concepts of closure and interior by intuitionistic fuzzy minimal semi-preopen set and intuitionistic fuzzy minimal semi-preclosed set and we introduce the concept of intuitionistic fuzzy minimal semi-precontinuous .We investigate some characterizations.

Keyword: Intuitionistic Fuzzy Minimal Open Set, Intuitionistic Fuzzy Minimal semi-preopen set , Intuitionistic Fuzzy Minimal Semi-preclosed Set and Intuitionistic Fuzzy Minimal semi- precontinuous.

1.Introduction:

In 1965, L.Zadeh(Zadeh,1965) was the first to introduce the concept of fuzzy set . In 1968, Change(Change,1968) introduced the concept of fuzzy topology on set X by axiomatizing, a collection T of fuzzy subsets of X, In (Chattopadyay, et.al.,1992), introduced the concept of fuzzy topology redefined by a gradation of openness and investigated some fundamental properties, and obtained some properties of them. Atanassov introduced the concept of intuitionistic fuzzy set which is a generalization of fuzzy set in Zadeh's sense (Atanassov,1986). D¸ coker introduced the concept of intuitionistic fuzzy topological spaces (coker,1998) by using the intuitionistic fuzzy sets, which is an extended concept of fuzzy topological spaces in Chang's sense. In 2002, Mondal and Samanta introduced the concept of intuitionistic gradations of openness (Mondal and Samanta,2002) which is a generalization of the concept of gradation of openness defined by Chattopadyay.In 2011, B.M.Ittanag and R.S.Wali introduced a new class of sets called fuzzy minimal open stes and fuzzy maximal open sets in fuzzy topological spaces(Ittanag and Wali,2011).In 2014 Y.K.Kim introduced the concept of Fuzzy(r,s)-minimal semiopen sets and fuzzy (r,s)-minimal semicontinuous mappings on fuzzy(r,s)-minimal space(Kim and Min,2014) . In this paper, we introduce the concept of intuitionistic fuzzy minimal open sets on intuitionistic fuzzy minimal topology. We also introduce a intuitionistic fuzzy minimal semi-preopen .We introduce intuitionistic fuzzy minimal semi-preinterior operators, intuitionistic fuzzy minimal semi-precloure operators ,we study some basic properties for them.We also investigate characterizations for the concept of intuitionistic fuzzy minimal semi-precontinuous in terms of intuitionistic fuzzy minimal semi-preinterior operators and intuitionistic fuzzy minimal semi-preclosure operators.

2.Preliminaries:

Let X be a nonempty set ; $I = [0,1]$, the closed unit interval of real line;I_0=(0,1]; I_1=[0,1);

I^X will denote the set of all fuzzy sets of X. $\underline{0}$ and $\underline{1}$ will denote the characteristic functions of φ and X, respectively.

Definition 2.1 (Min andAbbas,2013):

An intuitionistic gradation of openness (IGO for short) on set X an order pair (T, T^*) of mapping from I^X to I such that:

(IGO1) $T(A) + T^*(A) \leq 1, \forall A \in I^X$,

(IGO2) $T(\underline{0}) = T(\underline{1}) = 1, T^*(\underline{0}) = T^*(\underline{1}) = 0$,

(IGO3) $T(A_1 \wedge A_2) \geq T(A_1) \wedge T(A_2)$ *and* $T^*(A_1) \wedge (A_2) \leq T^*(A_1) \vee T^*(A_2)$ for each $A_i \in I^X, i = 1$

(IGO4) $T(\vee_{i\in\Gamma} A_i) \geq \wedge_{i\in\Gamma} T(A_i)$ *and* $T^*(\vee_{i\in\Gamma} A_i) \leq \vee_{i\in\Gamma} T^*(A_i)$ for each $A_i \in I^X, i \in \Gamma$.

The triple(X, T, T^*) is called an intuitionistic fuzzy topological space (IFTS for short). T and T^*may be interpreted as gradation of openness and gradation of non-openness, respectively.

Definition 2.2(Atanassov,1986):

Let X be a nonempty set and the IFSs A and B be of the form A=$\{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$,B=$\{< x, \mu_B(x), \gamma_B(x) >: x \in X\}$,Then

1. A≤B iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$
2. A=B iff A≤B and B≤A.
3. A∧B= $\{< x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) >: x \in X\}$.
4. A∨B=$\{< x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) >: x \in X\}$.

Definition 2.3:

A non empty fuzzy open set $\mu(\neq \underline{1})$,is said to be intuitionistic fuzzy minimal open set (briefly IFMOs) if any intuitionistic fuzzy open set which is contained in μ is either $\underline{0}$ or μ .

Example: 2.4:

Let $X= \{a, b, c\}$, define subsets $\mu_1, \mu_2 \in I^X$ as follows:

$\mu_1(a) = 0.5$ $\mu_1(b) = 0.3$ $\mu_1(c) = 0.6$

$\mu_2(a) = 0.3$ $\mu_2(b) = 0.4$ $\mu_2(c) = 0.3$

$$T(\lambda) = \begin{cases} 1 & if \lambda = \underline{0}, \underline{1} \\ 1/2 & if \lambda = \mu_1 \\ 1/3 & if \lambda = \mu_2 \\ 0 & otherwise \end{cases}$$

$$T^*(\lambda) = \begin{cases} 0 & if \lambda = \underline{0}, \underline{1} \\ 1/2 & if \lambda = \mu_1 \\ 2/3 & if \lambda = \mu_2 \\ 1 & otherwise \end{cases}$$

Then (T, T^*) is an intuitionistic gradation of openness let r=$1/2$ and s=$2/3$ then μ_2 is $IfMOs$.

Definition 2.5:

Let (X, T, T^*) be intuitionistic fuzzy topological space(for short IFTS) define an operator $C_{T,T^*}: I^X \times I_0 \times I_1 \rightarrow I^X$ by:

$C_{\mathcal{M}}(\lambda, r, s) = \wedge \{\mu \in I^X: \lambda \leq \mu, \underline{1} - \mu \in IFMOs\}$.

$I_{\mathcal{M}}(\lambda, r, s) = \vee \{\mu \in I^X: \lambda \geq \mu, \mu \in IFMOs\}$.

Theorem 2.6 :

let (X, T, T^*)be IFTS and $\lambda, \mu \in I^X$,then:

(1) $I_{\mathcal{M}}(\lambda, r, s) \leq \lambda$ and if $\lambda \in$ IFM set then $I_{\mathcal{M}}(\lambda, r, s) = \lambda$.

(2) $C_{\mathcal{M}}(\lambda, r, s) \geq \lambda$ and $\underline{1} - \lambda \in$ IFM set then $C_{\mathcal{M}}(\lambda, r, s) = \lambda$.

(3) If $\lambda \leq \mu$ *then* $I_{\mathcal{M}}(\lambda, r, s) \leq I_{\mathcal{M}}(\mu, r, s)$ and$C_{\mathcal{M}}(\lambda, r, s) \geq C_{\mathcal{M}}(\mu, r, s)$.

(4) $I_{\mathcal{M}}(\lambda \wedge \mu, r, s) = I_{\mathcal{M}}(\lambda, r, s) \wedge I_{\mathcal{M}}(\mu, r, s)$ and $C_{\mathcal{M}}(\lambda \vee \mu, r, s) = C_{\mathcal{M}}(\lambda, r, s) \vee C_{\mathcal{M}}(\mu, r, s)$.

(5) $I_{\mathcal{M}}(I_{\mathcal{M}}(\mu, r, s), r, s) = I_{\mathcal{M}}(\lambda, r, s)$and $C_{\mathcal{M}}(C_{\mathcal{M}}(\mu, r, s), r, s) = C_{\mathcal{M}}(\lambda, r, s)$.

$\underline{1} - C_{\mathcal{M}}(\lambda, r, s) = I_{\mathcal{M}}(\underline{1} - \lambda, r, s)$ and $\underline{1} - I_{\mathcal{M}}(\lambda, r, s) = C_{\mathcal{M}}(\underline{1} - \lambda, r, s)$.

Definition 2.7:

let $(X,T,T^*)$and (Y,η,η^*) be two IFTSs. then $f:(X,T,T^*)\to(Y,\eta,\eta^*)$ is said to be IFM-continuous mapping if $f^{-1}(\mu)$ is intuitionistic fuzzy open set in X for each open set $\mu\in I^Y$.

Theorem2.8 :

let $f:(X,T,T^*)\to(Y,\sigma,\sigma^*)$be afunction

(1) $f$is IFM-continuos.

(2) $\underline{1}-f^{-1}(\mu)\in T$,for each $\underline{1}-\mu\in\sigma$.

(3) $f(C_{\mathcal{M}}(\lambda,r,s))\le C_{\mathcal{M}}(f(\lambda),r,s)$,for $\lambda\in I^X$.

(4) $C_{\mathcal{M}}(f^{-1}(\mu),r,s)\le f^{-1}\big(C_{\mathcal{M}}(\mu,r,s)\big)$,for $\mu\in I^Y$.

$f^{-1}\big(I_{\mathcal{M}}(\mu,r,s)\big)\le I_{\mathcal{M}}(f^{-1}(\mu),r,s)$, for $\mu\in I^Y$.

Then (1) ⇔ (2) ⟹ (3) ⇔ (4) ⇔ (5).

3. Intuitionistic Fuzzy Minimal Semi-preopen Set and Intuitionistic Fuzzy Minimal Semi-precontinuity

Definition 3.1:

Let (X,T,T^*) be an intuitionistic fuzzy topological space,

1)) A fuzzy set $\lambda\in I^X$ is said to be an intuitionistic fuzzy minimal semi- preopen set if and only if there exist $r\in I_0, s\in I_1$

Such that:

$\lambda\le C_{\mathcal{M}_{T,T^*}}(I_{\mathcal{M}_{T,T^*}}(C_{\mathcal{M}_{T,T^*}}(\lambda,r,s),r,s),r,s)$

2) A fuzzy set $\lambda\in I^X$ is said to be an intuitionistic fuzzy minimal semi-preclosed set if and only if there exist $r\in I_0, s\in I_1$

Such that:

$I_{\mathcal{M}_{T,T^*}}(C_{\mathcal{M}_{T,T^*}}\left(I_{\mathcal{M}_{T,T^*}}(\lambda,r,s),r,s\right),r,s)\le\lambda$.

Example 3.2:

Let $X=\{a,b,c\}$ and $\mu_1,\mu_2,\mu_3\in I^X$ defined as follows:

$\mu_1(a)=0.4$ $\mu_1(b)=0.6$ $\mu_1(c)=0.3$

$\mu_2(a)=0.4$ $\mu_2(b)=0.3$ $\mu_2(c)=0.3$

$\mu_3(a)=0.6$ $\mu_3(b)=0.4$ $\mu_3(c)=0.7$

We define $T,T^*:I^X\to I$ as follows:

$$T(\lambda)=\begin{cases}1 & if\ \lambda=\underline{0},\underline{1}\\ 1/2 & if\ \lambda=\mu_1\\ 0 & \text{otherwise}\end{cases}\qquad T^*(\lambda)=\begin{cases}0 & if\ \lambda=\underline{0},\underline{1}\\ 1/2 & if\ \lambda=\mu_1\\ 1 & \text{otherwise}\end{cases}$$

otherwise

Then(X,T,T^*) is an intuitionistic fuzzy topological space. let $r=1/2$ and $s=1/2$ then,$\mu_2$is an intuitionistic fuzzy minimal semi-preopen open set.

Theorem 3.3:

Let (X,T,T^*) is IFTS. If μ_i are IF minimal semi-preopen sets,then

$\vee\mu_i$ is IF minimal semi-preopen set.

Proof : Let μ_i is IF minimal semi-preopen set for i∈J .then since $\mu_i\le\vee\mu_i$,

$$\mu_i\le C_{\mathcal{M}_{T,T^*}}\left(I_{\mathcal{M}_{T,T^*}}(C_{\mathcal{M}_{T,T^*}}(\mu_i,r,s),r,s),r,s\right)$$
$$\le C_{\mathcal{M}_{T,T^*}}(I_{\mathcal{M}_{T,T^*}}(C_{\mathcal{M}_{T,T^*}}(\vee\mu_i,r,s),r,s),r,s)$$

This implies $\vee\mu_i\le C_{\mathcal{M}_{T,T^*}}(I_{\mathcal{M}_{T,T^*}}(C_{\mathcal{M}_{T,T^*}}(\vee\mu_i,r,s),r,s),r,s)$

So $\vee\mu_i$ is IF minimal semi-preopen set.

Definition 3.4:

Let (X,T,T^*) is IFTS. For $\lambda\in I^X, r\in I_0, s\in I_1$

$C_{\mathcal{M}sp}(\lambda,r,s)=\wedge\{\mu\in I^X:\lambda\le\mu,\mu\in IFminimal\ semi-preclosed\ set\}$.

$I_{\mathcal{M}sp}(\lambda,r,s)=\vee\{\mu\in I^X:\mu\ge\lambda,\mu\in IF\ nimimalsemi-pre\ open\ sets\}$.

Theorem 3.5:

Let (X,T,T^*) is IFTS. For $\lambda\in I^X, r\in I_0, s\in I_1$,then:

1. $I_{\mathcal{M}sp}(\lambda,r,s)\le\lambda$.
2. λ is $IFminimal\ semi-preopen$ iff $I_{\mathcal{M}sp}(\lambda,r,s)=\lambda$.
3. If $\lambda\le\mu$,then $I_{\mathcal{M}sp}(\lambda,r,s)\le I_{\mathcal{M}sp}(\mu,r,s)$.

4. $I_{\mathcal{M}sp}\left(I_{\mathcal{M}sp}(\lambda,r,s),r,s\right) = I_{\mathcal{M}sp}(\lambda,r,s)$.
5. $C_{\mathcal{M}sp}\left(\underline{1}-\lambda,r,s\right) = \underline{1} - I_{\mathcal{M}sp}(\lambda,r,s)$ and $I_{\mathcal{M}sp}\left(\underline{1}-\lambda,r,s\right) = \underline{1} - C_{\mathcal{M}sp}(\lambda,r,s)$.

Proof: (1),(2),(3),(4) are obtain from theorem2.
(5)For$\lambda \in I^X, r \in I_0, s \in I_1, \underline{1} - I_{\mathcal{M}sp}(\lambda,r,s) = \underline{1} - \vee \{\mu \in I^X : \mu \geq \lambda, \mu \in IF\ nimimalsemi - pre\ open\ sets\}$
$= \wedge \{\underline{1} - \mu \in I^X : \lambda \leq \mu, \mu \in IFminimal\ semi - preclosed\ set\}$
$= \wedge \{\underline{1} - \mu \in I^X : \underline{1} - \lambda \leq \underline{1} - \mu, \mu \in IFminimal\ semi - preclosed\ set\}$
$= C_{\mathcal{M}sp}\left(\underline{1}-\lambda,r,s\right)$
Similarly, we have $I_{\mathcal{M}sp}\left(\underline{1}-\lambda,r,s\right) = \underline{1} - C_{\mathcal{M}sp}(\lambda,r,s)$.

Theorem 3.6 :
Let (X,T,T^*) is IFTS. For $\lambda \in I^X, r \in I_0, s \in I_1$,then:
1. $\lambda \leq C_{\mathcal{M}sp}(\lambda,r,s)$.
2. If $\lambda \leq \mu$, then $C_{\mathcal{M}sp}(\lambda,r,s) \leq C_{\mathcal{M}sp}(\mu,r,s)$.
3. λ is $IFminimal\ semi - preclos$ iff $C_{\mathcal{M}sp}(\lambda,r,s) = \lambda$.
4. $C_{\mathcal{M}sp}\left(C_{\mathcal{M}sp}(\lambda,r,s),r,s\right) = C_{\mathcal{M}sp}(\lambda,r,s)$.

Proof : It is similar to the proof of theorem 3.5.

Definition 3.7:
Let $(X,T,T^*)$and (Y,σ,σ^*)be two IFTSs. then a mapping $f: X \to Y$is said to be IFM semi-precontinuos if for every IF minimal open set $\mu \in I^Y$,$f^{-1}(\mu)$ is IFM semi-preopen set.
Every IFM-continuos mapping is IFM semi-precontinuos but the converse is not true in general.

Example 3.8:
Let $X = \{a,b,c\}\ and\ \mu_1,\mu_2 \in I^X$ defined as follows:
$\mu_1(a) = 0.5$ $\quad \mu_1(b) = 0.3$ $\quad \mu_1(c) = 0.6$
$\mu_2(a) = 0.3$ $\quad \mu_2(b) = 0.2$ $\quad \mu_2(c) = 0.6$
Define intuitionistic gradation of openness $(T_1,{T_1}^*)\ and\ (T_2,{T_2}^*): I^X \to I$

$$T(\lambda) = \begin{cases} 1 & if\ \lambda = \underline{0},\underline{1} \\ 1/3 & if\ \lambda = \mu_1 \\ 1/2 & if\ \lambda = \mu_2 \\ 2/3 & if\ \lambda = \mu_1 \wedge \mu_2 \\ 1/3 & if\ \lambda = \mu_1 \vee \mu_2 \\ 0 & \text{otherwise} \end{cases} \qquad T_1^*(\lambda) = \begin{cases} 0 & if\ \lambda = \underline{0},\underline{1} \\ 2/3 & if\ \lambda = \mu_1 \\ 1/2 & if\ \lambda = \mu_2 \\ 1/3 & if\ \lambda = \mu_1 \wedge \mu_2 \\ 2/3 & if\ \lambda = \mu_1 \vee \mu_2 \\ 1 & \text{otherwise} \end{cases}$$

$$T_2(\lambda) = \begin{cases} 1 & if\ \lambda = \underline{0},\underline{1} \\ 1/3 & if\ \lambda = \mu_2 \\ 0 & \text{otherwise} \end{cases} \qquad T_2^*(\lambda) = \begin{cases} 0 & if\ \lambda = \underline{0},\underline{1} \\ 2/3 & if\ \lambda = \mu_2 \\ 1 & \text{otherwise} \end{cases}$$

otherwise
Let r $=1/3$and s$=2/3$ $f:(X,T_1,{T_1}^*) \to (X,T_2,{T_2}^*)$ IFM semi-precontinuous but the identity mapping not IF continuous .

Theorem 3.9 :
Let $(X,T,T^*),(Y,\eta,\eta^*)$ be two intuitionistic fuzzy minimal topological spaces, and $f:(X,T,T^*) \to (Y,\eta,\eta^*)$ be a mapping,the following statements are equivalent:
1. f is IFM semi-precontinuous.
2. $f^{-1}(\mu)$ is IFM semi-preclosed set for each IFM closed set $\mu \in I^Y$.
3. $f\left(C_{\mathcal{M}spT,T^*}(\lambda,r,s)\right) \leq C_{\mathcal{M}sp\eta,\eta^*}(f(\lambda),r,s)$,for $\lambda \in I^X$.
4. $C_{\mathcal{M}spT,T^*}(f^{-1}(\mu),r,s) \leq f^{-1}\left(C_{\mathcal{M}sp\eta,\eta^*}(\mu,r,s)\right)$ for $\mu \in I^Y$.

$f^{-1}\left(I_{\mathcal{M}sp\eta,\eta^*}(\mu,r,s)\right) \leq I_{\mathcal{M}spT,T^*}(f^{-1}(\mu),r,s)$ for $\mu \in I^Y$.

Proof: (1)⟹(2) It is obvious .(2) ⟹(3) for $\lambda \in I^X$
$f^{-1}(C_{\mathcal{M}sp\eta,\eta^*}(f(\lambda),r,s)) = f^{-1}(\wedge \{\mu \in I^Y : f(\lambda) \leq \mu\ and\ \mu\ is\ IFM\ closed\}$
$= \wedge \{f^{-1}(\mu) \in I^Y : \lambda \leq f^{-1}(\mu) and\ f^{-1}(\ \mu) is\ IFMsemi - preclosed\}$
$= C_{\mathcal{M}spT,T^*}(\lambda,r,s)$
Hence $f\left(C_{\mathcal{M}spT,T^*}(\lambda,r,s)\right) \leq C_{\mathcal{M}sp\eta,\eta^*}(f(\lambda),r,s)$.
(3) ⟹(4) for $\mu \in I^Y$

$f(C_{\mathcal{M}spT,T^*}(f^{-1}(\mu),r,s)) \leq C_{\mathcal{M}sp\eta,\eta^*}(f(f^{-1}(\mu),r,s) \leq C_{\mathcal{M}sp}(\mu,r,s)$.
Thus we have $C_{\mathcal{M}spT,T^*}(f^{-1}(\mu),r,s) \leq f^{-1}\left(C_{\mathcal{M}sp\eta,\eta^*}(\mu,r,s)\right)$.
(3) ⟹(4) for $\mu \in I^Y$
$f^{-1}\left(I_{\mathcal{M}sp\eta,\eta^*}(\mu,r,s)\right) = f^{-1}\left(\underline{1} - C_{\mathcal{M}sp\eta,\eta^*}(\underline{1} - \mu,r,s)\right)$
$=\underline{1} - f^{-1}\left(C_{\mathcal{M}sp\eta,\eta^*}(\underline{1}-\mu,r,s)\right) \leq \underline{1} - C_{\mathcal{M}spT,T^*}\left(f^{-1}(\underline{1}-\mu),r,s\right)$
$= I_{\mathcal{M}spT,T^*}(f^{-1}(\mu),r,s)$ .Hence $f^{-1}\left(I_{\mathcal{M}sp\eta,\eta^*}(\mu,r,s)\right) \leq I_{\mathcal{M}spT,T^*}(f^{-1}(\mu),r,s)$.
(5) ⟹(1)let λ be any IFM open set. Then from (5),it follows $f^{-1}(\lambda) = f^{-1}(I_{\mathcal{M}sp\eta,\eta^*}(\lambda,r,s))$
$\leq I_{\mathcal{M}spT,T^*}(f^{-1}(\lambda),r,s)$ and $f^{-1}(\lambda) = I_{\mathcal{M}spT,T^*}(f^{-1}(\lambda),r,s)$ .this implies $f^{-1}(\lambda)$is IFM semi-preopen set .Hence f is IFM semi-precontinuous.

Theorem 3.10:

Let (X,T,T^*) be an IFT and $\lambda \in I^X$.Then:
1.$I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}(\lambda,r,s),r,s\right),r,s\right) \leq I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}(C_{\mathcal{M}sp}(\lambda,r,s),r,s),r,s\right),r,s\right) \leq C_{\mathcal{M}sp}(\lambda,r,s)$
2. $I_{\mathcal{M}sp}(\lambda,r,s) \leq C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}(I_{\mathcal{M}sp}(\lambda,r,s),r,s),r,s\right),r,s\right)$
$\leq C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}(I_{\mathcal{M}}(\lambda,r,s),r,s),r,s\right),r,s\right.$.
Proof:(1) since $C_{\mathcal{M}sp}(\lambda,r,s)$ is IFM semi-preclosed, it obtain from definition(3.4) and theorem (3.6) .
(2)Obvious.

Theorem 3.11:

Let $(X,T,T^*),(Y,\eta,\eta^*)$ be two intuitionistic fuzzy minimal topological spaces, and $f:(X,T,T^*) \to (Y,\eta,\eta^*)$ be a mapping, the following statements are equivalent:

1. f is IFM semi-precontinuous.
2. $f^{-1}(\mu) \leq C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}(\mu,r,s),r,s\right),r,s\right)$, for each IFM open set μ in Y.
3. $I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}(f^{-1}(\beta),r,s),r,s\right),r,s\right) \leq f^{-1}(\beta)$, for each IFM closed set β in Y.
4. $f\left(I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}(\lambda,r,s),r,s\right),r,s\right)\right) \leq C_{\mathcal{M}\eta,\eta^*}(f(\lambda,r,s))$,for $\lambda \in I^X$.
5. $C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}(f^{-1}(\mu),r,s),r,s\right),r,s\right) \leq f^{-1}(C_{\mathcal{M}\eta,\eta^*}(\mu,r,s))$, for $\mu \in I^Y$.
6. $f^{-1}\left(I_{\mathcal{M}\eta,\eta^*}(\mu,r,s)\right) \leq C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}(f^{-1}(\mu),r,s),r,s\right),r,s\right)$.

Proof: (1)⟺(2)It is easily obtain from concepts of IFM semi-precontinuity and IFM semi-preopen sets.
(1)⟺(3)Obvious.
(1)⟺(4) for $\lambda \in I^X$,we have $\left(I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}(\lambda,r,s),r,s\right),r,s\right) \leq C_{\mathcal{M}T,T^*}(\lambda,r,s)\right.$
$\leq f^{-1}(f\left(C_{\mathcal{M}T,T^*}(\lambda,r,s)\right)) \leq f^{-1}(C_{\mathcal{M}\eta,\eta^*}(f(\lambda),r,s))$
So
$f\left(I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}(\lambda,r,s),r,s\right),r,s\right) \leq C_{\mathcal{M}\eta,\eta^*}(f(\lambda),r,s)\right.$.
(4)⟹(5) Obvious.
(5)⟹(6)For $\mu \in I^Y$,from hypothesis,
$f^{-1}\left(I_{\mathcal{M}\eta,\eta^*}(\mu,r,s)\right) = f^{-1}(\underline{1} - (C_{\mathcal{M}\eta,\eta^*}((\underline{1} - \mu),r,s))$
$= \underline{1} - f^{-1}(C_{\mathcal{M}\eta,\eta^*}((\underline{1}-\mu),r,s))$
$\leq \underline{1} - I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}(f^{-1}(\underline{1} - \mu),r,s\right),r,s\right),r,s)$
$= C_{\mathcal{M}T,T^*}\left(I_{\mathcal{M}T,T^*}\left(C_{\mathcal{M}T,T^*}(f^{-1}(\mu),r,s\right),r,s\right),r,s)$
So we have (6).
(6)⟹(1) For $\mu \in I^Y$,let μ be IFM open set. Then since

$\mu = I_{\mathcal{M}_{\eta,\eta^*}}(\mu, r, s)$, by hypothesis
$f^{-1}(\mu) = f^{-1}(I_{\mathcal{M}_{\eta,\eta^*}}(\mu, r, s))$
$\leq C_{\mathcal{M}_{T,T^*}}\left(I_{\mathcal{M}_{T,T^*}}\left(C_{\mathcal{M}_{T,T^*}}(f^{-1}(\mu), r, s\right), r, s\right), r, s)$
And so $f^{-1}(\mu)$ is IFM semi-preopen set.Hence f is IFM sem-precontinuous.

Corollary3.12: Let (X, T, T^*) *and* (Y, η, η^*) be two intuitionistic fuzzy topological spaces and $f: X \to Y$ be a mapping. For each $\lambda \in I^X, r \in I_0, s \in I_1$, then the following statements are equivalent:
1) f is an intuitionistic fuzzy minimal semi-precontinuous mapping.
2) $f(\, I_{\mathcal{M}T,T^*}(\lambda, r, s)) \leq I_{\mathcal{M}n,n^*}(C_{\mathcal{M}spn,n^*}(f(\lambda)), r, s)$.

4.References:

Atanassov .K.T., Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986),87-96.

Chang .C. L., Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968), 182–190.

Chattopadyay .K.C., Hazar .R.N. and Samanta .S.K., Gradation of openness: fuzzy topology, fuzzy sets and systems49(1992),237-242.

Coker .D., An introduction to intuitionistic fuzzy topological spaces,Fuzzy Sets and Systems 88(1997),81-89.

Kim .Y.K., Min W.K., Fuzzy(r,s)-minimal semiopen sets and fuzzy (r,s)-m semicontinuous mappings on fuzzy(r,s)-minimal space, International Journal of pure and Applied Mathematics,Vol.90,No.3(2014),309-320.

Min .W.K., Abbas. S.E.,(r,s)-fuzzy minimal structures and (r,s)-fuzzy minimal spaces, Journal of Intelligent and fuzzy systems,doi:10(2013).3233/IFS-120669.

Ittanagi .B.M. and Wali. R.S., On fuzzy minimal open and fuzzy maximal open sets in fuzzy topologyical spaces, Int .J. of Mathematical Sciences and Applications, Vol.1,No.2,(2011) 1024-1037.

Mondal .T.K. and Samanta .S.K., On intuitionistic gradation of openness: fuzzy topology, fuzzy sets and systems131(2002),323-336.

Sostak .A.P., On a fuzzy topological structure, Suppl . Rend . Circ . Matem . Palerms ser II,11(1985),89-103.

Zadeh .L. A., Fuzzy sets, Inform. and Control 8 (1965), 338–35.