Comparative study to some semi numerical methods for solving nonlinear Wave-like Equations with variable coefficient

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Abstract:

In this paper we solve the nonlinear wave-like Equations with variable coefficient by three Semi numerical methods, varational iteration method (VIM), homotopy perturbation method (HPM) and homotopy analysis method (HAM). The solution obtained by these methods are Compared with the solution obtained by Adomian decomposition method (ADM) and the exact solution [8].

Keywords: ADM, VIM, HPM, HAM, Nonlinear wave-like Equations with variable coefficient.

1. Introduction:

Recently there are four famous methods for solving nonlinear problems, ADM, VIM, HPM and HAM. In this section we will give short review for these methods. We start with the oldest one, it is ADM, which discovered firstly by Adomian [2, 3] in the early eighties of the last century. The method was applied for a wide class of nonlinear problems. The convergence of ADM discussed by Abbaoui and Cherruault [1].

VIM introduced by He [5] the method was impressive in applications, and it is easy in computations. Also He introduced HPM and proved its Convergence [6], it was a practical method for solving nonlinear problems and a wonderful application for homotopy ideas.

Finally, Liao [7] introduced the HAM, It is used successfully for solving many nonlinear problems, it is also based on homotopy method, also Liao proved the convergence of HAM [7]. These methods are semi numerical methods because they are produced series solution. In this paper, we apply VIM, HPM and HAM for solving the nonlinear wave-like Equations with variable coefficient [8], in the following form,

$$u_{tt} = \sum_{i,j}^{n} F_{1ij}(X,t,u)\frac{\partial^{k+m}}{\partial x_i^k \partial x_j^m} F_{2ij}\left(u_{x_i}, u_{x_j}\right) + \sum_{i,j}^{n} G_{1i}(X,t,u)\frac{\partial^p}{\partial x_i^p} G_{2i}\left(u_{x_i}\right)$$
$$+ H(X,t,u) + S(X,t) \tag{1}$$

with the initial conditions $\qquad u(X,0) = a_0(X) \qquad , \qquad u_t(X,0) = a_1(X).$

Here $\quad X = (x_1, x_2, \ldots, x_n), \quad F_{1ij}$ and $G_{1i} \quad$ are nonlinear functions of $\quad X$, t and u. F_{2ij} and G_{2i} are nonlinear functions of derivatives of $\quad x_i \quad$ and $\quad x_j, \quad$ whilst H and S are nonlinear functions. k, m, p are integers, and compared the result with the exact solution and ADM solution [8]. In the next three sections we introduced the procedures for these methods.

To apply VIM, HPM and HAM for Nonlinear wave-like Equation with variable coefficient, we write Equation (1) in the operator form as;

$$L(u(X,t)) = N(u(X,t)) + S(X,t) \tag{2}$$

where

$$N(u) = \sum_{i,j}^{n} F_{1ij}(X,t,u)\frac{\partial^{k+m}}{\partial x_i^k \partial x_j^m} F_{2ij}\left(u_{x_i}, u_{x_j}\right) + \sum_{i,j}^{n} G_{1i}(X,t,u)\frac{\partial^p}{\partial x_i^p} G_{2i}\left(u_{x_i}\right) + H(X,t,u)$$

and $\qquad L(u) = \dfrac{\partial^2 u}{\partial t^2}$.

We use Maple 13 software for computations.

2. VIM for nonlinear wave-like Equation:

In this section, a description of the VIM is given to handle the nonlinear problem (2). According to He's VIM, we can construct a correction functional as follows,

$$u_{n+1}(X,t) = u_n(X,t) + \int_0^t \lambda(s)\left(Lu_n(X,s) - N\tilde{u}_n(X,s) - S(X,s)\right) ds \tag{3}$$

where λ is a general Lagrange multiplier which can be identified optimally via variational theory. Here $\tilde{u}_n$ is considered as a restricted variation which means $\delta\tilde{u}_n = 0$. Therefore, we first determine the Lagrange multiplier λ that will be identified optimally via integration by parts. The successive approximation $u_n(X,t)$, $n \geq 0$ of the solution $u(X,t)$ will be readily obtained upon using the obtained Lagrange multiplier and by using any selective function u_0. The zeroth approximation u_0 may be selected any

function that just satisfies, at least, the initial and boundary conditions. With λ determined, then several approximations $u_n(X,t)$, $n \geq 0$ follow immediately. Consequently, the exact solution may be obtained as, $u(X,t) = \underset{n \to \infty}{Lim}\, u_n(X,t)$.

3. HPM for nonlinear wave-like Equation:

In this section, we shall demonstrate the application of HPM to solve Equation (2). By the homotopy perturbation method, we construct a homotopy as $v(r,p) = \Omega \times [0,1] \to R$, Which satisfy the following Equation:

$$H(v,p) = (1-p)\left[L(v) - L(u_0)\right] + p\left[L(v) + N(v)\right] = 0$$

or,
$$H(v,p) = L(v) - L(u_0) + p\left[L(u_0) + N(v)\right] = 0 \qquad (4)$$

Where $p \in [0,1]$ the embedding parameter and u_0 the initial approximation satisfying the boundary conditions. Now from Equation (4) we have:

$H(v,0) = L(v) - L(u_0) = 0$ and $H(v,1) = L(v) + N(v) = 0$.

In topology this is called deformation, also $H(v,0) = L(v) - L(u_0)$ and $H(v,1) = L(v) + N(v)$ are called homotopic. By the homotopy perturbation theory, we can first use the embedding parameter p as a small parameter and assume that nth solution of Equation (2) can be written as a power series in p as followed:

$$v = \sum_{n=0}^{\infty} p^n v_n \qquad (5)$$

Setting $p = 1$ we have the approximate solution of Equation (2) in following form:

$$u(X,t) = \underset{p \to 1}{Lim}\, v = \sum_{n=0}^{\infty} v_n \qquad (6)$$

4. HAM for nonlinear wave-like Equation:

In order to show the basic idea of HAM, we rewrite problem (2) in the following form,

$$N(u(X,t)) = u_{tt}(X,t) - \sum_{i,j}^{n} F_{1ij}(X,t,u)\frac{\partial^{k+m}}{\partial x_i^k \partial x_j^m} F_{2ij}\left(u_{x_i}, u_{x_j}\right) - \sum_{i,j}^{n} G_{1i}(X,t,u)\frac{\partial^p}{\partial x_i^p} G_{2i}\left(u_{x_i}\right)$$
$$- H(X,t,u) - S(X,t)$$

where N is a nonlinear operator. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of the HAM, we first construct the so called zeroth-order deformation Equation

$$(1-q)L[\phi(X,t;q)-u_0(X,t)]=qhH(X,t)N[\phi(X,t;q)] \qquad (7)$$

where q is the embedding parameter, $h \neq 0$ is an auxiliary parameter, $H(X,t)$ non zero auxiliary function, L is an auxiliary linear operator, $\phi(X,t;q)$ is an unknown function and $u_0(X,t)$ is an initial solution obtaining from the initial conditions and the remaining term. It is obvious that when the embedding parameter $q=0$ and $q=1$, Equation (7) becomes, $\phi(X,t;0)=u(X,0)$, $\phi(X,t;1)=u(X,t)$ respectively. Thus as q increases from 0 to 1, the solution $\phi(X,t;q)$ varies from the initial guess $u_0(X,t)$ to the solution $u(X,t)$. Expanding $\phi(X,t;q)$ in Taylor series with respect to q, one has

$$\phi(X,t;q)=u_0(X,t)+\sum_{k=1}^{\infty}u_k(X,t)q^k \quad \text{where} \quad u_k(X,t)=\frac{1}{k!}\frac{\partial^k \phi(X,t;q)}{\partial q^k}\bigg|_{q=0} .$$

The convergence of the above series depends upon the auxiliary parameter h. If it is convergent at $q=1$, we have, $u(X,t)=u_0(X,t)+\sum_{k=1}^{\infty}u_k(X,t)$ which must be one of the solutions of the original nonlinear Equation, as proven by Liao [7].

Define the vectors $\vec{u}_n(X,t)=\{u_0(X,t),u_1(X,t),...,u_n(X,t)\}$

Differentiating the zeroth-order deformation Equation k-times with respect to q and then dividing them by $k!$ and finally setting $q=0$, we get the following kth-order deformation Equation:

$$L[u_k(X,t)-\chi_k u_{k-1}(X,t)]=H(X,t)hR_k(\vec{u}_{k-1})$$

where $R_k(\vec{u}_{k-1})=\frac{1}{(k-1)!}\frac{\partial^{k-1}N[\phi(X,t;q)]}{\partial q^{k-1}}\bigg|_{q=0}$ and $\chi_k=\begin{cases}0, & k \leq 1 \\ 1, & k > 1\end{cases}$

It should be emphasized that $u_k(X,t)$ for $k \geq 1$ is governed by the linear kth-order deformation Equation with linear boundary conditions that come from the original problem. Note that we define the linear operator as,

$$L[\phi(X,t;q)]=\frac{\partial^2 \phi(X,t;q)}{\partial t^2} , \text{ with the property } L[C_1(X)+C_2(X)t]=0 , \text{ where } C_1, C_2 \text{ are integral}$$

constants.

Liao [7], proved that the ADM is a special case of HAM under the assumptions $u_0(X,t)=L^{-1}S(X,t)$, $H(X,t)=1$ and $h=-1$. In HAM we have a great freedom to choose the initial guess $u_0(X,t)$ and the auxiliary function $H(X,t)$, also the auxiliary parameter h provides us with a simply way to adjust and control convergence region and rate of solution series.

5. Test problems:

In this section, we apply VIM, HPM and HAM for two test problems [8], and compared the approximate analytical solution obtained for the nonlinear wave-like problems by the three methods with solutions obtained by ADM [8] and the exact solutions. We use six terms for the series solution for both problems.

5.1. problem 1.

Let us consider the 2-dimensional nonlinear wave-like Equation with variable coefficients

$$u_{tt} = \frac{\partial^2}{\partial x \partial y}\left(u_{xx}u_{yy}\right) - \frac{\partial^2}{\partial x \partial y}\left(x y u_x u_y\right) - u \tag{8}$$

With the initial conditions $u(x, y,0)=e^{xy}$ and $u_t(x, y,0)=e^{xy}$

The exact solution is $u(x, y,t)=e^{xy}(\sin t + \cos t)$

5.1.1. VIM

We assume that the zeroth term is $u_0(x, y,t)=e^{xy+t}$, also we set $\lambda = s - t$ [2], so we have,

$$u_{n+1}(x, y,t)=u_n(x, y,t)+\int_0^t (s-t)\left(Lu_n(x, y,s)-N\tilde{u}_n(x, y,s)-u(x, y,s)\right)ds$$

Hence we have the following solutions,

For $n=0$, $u_1 = e^{xy}\left(2-e^t+2t\right)$.

For $n=1$, $u_2 = e^{xy}\left(e^t - t^2 - \frac{1}{3}t^3\right)$.

For $n=2$, $u_3 = e^{xy}\left(2-e^t+2 t+\frac{1}{12}t^4+\frac{1}{60}t^5\right)$.

For $n=3$, $u_4 = e^{xy}\left(e^t - t^2 -\frac{1}{3}t^3 -\frac{1}{360}t^6 -\frac{1}{2520}t^7\right)$.

And so on.

5.1.2. HPM

We apply HPM for problem (8) by using Equations (4 - 6), with $u_0 = e^{xy}(1+t)$. We get the following solutions,

$$u_0 = e^{xy}(1+t),$$

$$u_1 = -e^{xy}\left(\frac{1}{2}t^2 + \frac{1}{6}t^3\right),$$

$$u_2 = e^{xy}\left(\frac{1}{24}t^4 + \frac{1}{120}t^5\right),$$

$$u_3 = -e^{xy}\left(\frac{1}{720}t^6 + \frac{1}{5040}t^7\right),$$

$$u_4 = e^{xy}\left(\frac{1}{40320}t^8 + \frac{1}{362880}t^9\right).$$

$$\vdots$$

For $n = 6$, we have the following approximate solution for Equation (8),

$$u = e^{xy}\left(1+t-\frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 - \frac{1}{720}t^6 - \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \cdots\right)$$

5.1.3. HAM

We define the nonlinear operator as, $N(u) = u_{tt} - \dfrac{\partial^2}{\partial x \partial y}(u_{xx}u_{yy}) + \dfrac{\partial^2}{\partial x \partial y}(x\,y\,u_x u_y) + u$.

To solve Equation (8) by means of the HAM, the initial approximation will be,

$$u_0(x,y,t) = u(x,y,0) + t\frac{\partial u(x,y,0)}{\partial t}\bigg|_{t=0} = e^{xy}(1+t).$$

Using the above definition, and chose $H(x,y,t) = 1$ we construct the zeroth-order deformation Equation,

$$(1-q)L[\phi(x,y,t;q) - u_0(x,y,t)] = q\,h\,N[\phi(x,y,t;q)]$$

Therefore, $\phi(x,y,t;q) = u_0(x,y,t) + \sum\limits_{k=1}^{\infty} u_k(x,y,t)q^k$

where $u_k(x,y,t) = \dfrac{1}{k!}\dfrac{\partial^k \phi(x,y,t;q)}{\partial q^k}\bigg|_{q=0}.$

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So, for $q=1$ we have $u(x, y,t)=u_0(x, y,t)+\sum_{k=1}^{\infty}u_k(x, y,t)$

Now define the vectors $\vec{u}_n(x, y,t)=\{u_0(x, y,t),u_1(x, y,t), \dots ,u_n(x, y,t)\}$.

Differentiating the zeroth-order deformation Equation k-times with respect to q and then dividing them by $k!$ and finally setting $q=0$, we get the following kth-order deformation Equation: $L[u_k(x, y,t)- \chi_k u_{k-1}(x, y,t)]= h R_k(\vec{u}_{k-1})$, with the boundary conditions

$$u_k(x, y,0) = \frac{\partial u_k}{\partial t}\bigg|_{t=0} = 0 \quad \text{where} \quad \chi_k = \begin{cases} 0, & k \leq 1 \\ 1, & k >1 \end{cases}.$$

and the solution of the kth-order deformation Equation becomes,

$$u_k(x, y,t)= L^{-1}\left[h R_k(\vec{u}_{k-1}(x, y,t))+ \chi_k u_{k-1}(x, y,t)\right] \quad \text{for} \quad k =1,2,3,\dots$$

Now, we successively obtain,

$$u_1(x, y,t)= he^{xy}\left(\frac{1}{6}t^3 + \frac{1}{2}t^2 \right)$$

$$u_2(x, y,t)= he^{xy}\left(\frac{1}{2}t^2 + \frac{1}{6}t^3 + h\left(\frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 \right)\right)$$

$$u_3(x, y,t)= he^{xy}\left(\frac{1}{2}t^2 + \frac{1}{6}t^3 + 2h\left(\frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 \right)\right.$$
$$\left. + h^2\left(\frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{12}t^4 + \frac{1}{60}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 \right)\right)$$

$$u_4(x, y,t)= he^{xy}\left(\frac{1}{2}t^2 + \frac{1}{6}t^3 + 3h\left(\frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 \right)\right.$$
$$+ 3h^2\left(\frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{12}t^4 + \frac{1}{60}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 \right)$$
$$\left. + h^3\left(\frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{8}t^4 + \frac{1}{40}t^5 + \frac{1}{240}t^6 + \frac{1}{1680}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 \right)\right)$$

$$\vdots$$

The series solution for $n = 6$ is,

$$u = e^{xy}\left(1+t+\frac{1}{2}h(5+5h^3 +10h^2 +10h + h^4)t^2 +\frac{1}{6}h(5+5h^3 +10h^2 +10h+h^4)t^3 \right.$$
$$\left. +\frac{1}{24}h^2(20h + 4h^3 +10+15h^2)t^4 +\cdots \right)$$

As pointed by Liao [7], the auxiliary parameter h can be employed to adjust the convergence region of the homotopy analysis solution. To investigate the influence of h on the solution series, we plot the so called h-curve of $u_{tt}(0.5,0.5,0)$ obtained from the six terms of HAM approximation solution as showed in Figure (1). According to this h-curve, it is easy to discover the valid region of h which corresponds to the line segment nearly parallel to the horizontal axis. It is clear that the series of solutions for this case is convergent when $-1.4 < h < -0.6$. For this we chose $h = -0.9$.

The Maximum error for $x = y = 0.5$ showed in Figures (3-6), also tables (1-4) shows the absolute errors when x, y and t vary from 0.1 to 0.9 .

5.2. problem 2.

Consider the following one dimensional nonlinear wave-like Equation

$$u_{tt} = x^2\left[\frac{\partial}{\partial x}(u_x u_{xx}) - (u_{xx})^2\right] - u, \qquad 0 \le x \le 1 \quad , \quad t \ge 0 \qquad (9)$$

with the initial conditions $u(x,0) = 0$ and $u_t(x,0) = x^2$ and boundary conditions $u(0,t) = 0$ and $u(1,t) = \sin t$. The exact solution is $u(x,t) = x^2 \sin t$.

5.2.1. VIM

We start the iterations with $u_0(x, y, t) = x^2 t$ and $\lambda = s - t$. The procedure will continue with the following iterations,

$$u_{n+1}(x, y, t) = u_n(x, y, t) + \int_0^t (s-t)\left(Lu_n(x, y, s) - N\tilde{u}_n(x, y, s) - u(x, y, s)\right) ds$$

We obtain the following solutions for six terms.

For $n = 1$, $u_1 = x^2\left(t - \frac{1}{6}t^3\right)$.

For $n = 2$, $u_2 = x^2\left(t - \frac{1}{6}t^3 + \frac{1}{120}t^5\right)$.

For $n = 3$, $u_3 = x^2\left(t - \frac{1}{6}t^3 + \frac{1}{120}t^5 - \frac{1}{5040}t^7\right)$.

For $n = 4$, $u_4 = x^2\left(t - \frac{1}{6}t^3 + \frac{1}{120}t^5 - \frac{1}{5040}t^7 + \frac{1}{362880}t^9\right)$. And so on.

5.2.2. HPM

By applying HPM and using Equations (4 - 6), we have the following solutions,

$$u_0 = x^2 t,$$

$$u_1 = -\frac{1}{6}x^2 t^3,$$

$$u_2 = \frac{1}{120}x^2 t^5,$$

$$u_3 = -\frac{1}{5040}x^2 t^7,$$

$$u_4 = \frac{1}{362880}x^2 t^9,$$
$$\vdots$$

For $n = 6$, the approximate solution for Equation (9), will be as followed,

$$u = x^2\left(t - \frac{1}{6}t^3 + \frac{1}{120}t^5 - \frac{1}{5040}t^7 + \frac{1}{362880}t^9 + \cdots\right)$$

5.2.3. HAM

Define the nonlinear operator as $N(u) = u_{tt} - x^2\left[\frac{\partial}{\partial x}(u_x u_{xx}) - (u_{xx})^2\right] + u$.

The initial approximation will be,

$$u_0(x,t) = u(x,0) + t\frac{\partial u(x,0)}{\partial t}\bigg|_{t=0} = x^2 t.$$

Now, chose $H(x,t) = 1$, then the zeroth-order deformation Equation is,

$$(1-q)L[\phi(x,t;q) - u_0(x,t)] = q\,h\,N[\phi(x,t;q)]$$

So, $\phi(x,t;q) = u_0(x,t) + \sum_{k=1}^{\infty} u_k(x,t)q^k$ where $u_k(x,t) = \frac{1}{k!}\frac{\partial^k \phi(x,t;q)}{\partial q^k}\bigg|_{q=0}$.

In the similar way we have the following,

$$L[u_k(x,t) - \chi_k u_{k-1}(x,t)] = h R_k(\vec{u}_{k-1}),$$

with the boundary conditions $u_k(x,0) = \frac{\partial u_k(x,t)}{\partial t}\bigg|_{t=0} = 0$.

The solution of the kth-order deformation Equation becomes,

$$u_k(x,t) = L^{-1}\left[h R_k(\vec{u}_{k-1}) + \chi_k u_{k-1}(x,t) \right] \quad \text{for} \quad k = 1,2,3,\ldots$$

Then we have the following solutions,

$$u_1(x,t) = \frac{1}{6} h t^3 x^2$$

$$u_2(x,t) = \frac{1}{6} h x^2\left(t^3 + h\left(t^3 + \frac{1}{20} t^5 \right) \right)$$

$$u_3(x,t) = \frac{1}{6} h x^2\left(t^3 + 2h\left(t^3 + \frac{1}{20} t^5 \right) + h^2\left(t^3 + \frac{1}{10} t^5 + \frac{1}{840} t^7 \right) \right)$$

$$u_4(x,t) = \frac{1}{6} h x^2\left(t^3 + 3h\left(t^3 + \frac{1}{20} t^5 \right) + 3h^2\left(t^3 + \frac{1}{10} t^5 + \frac{1}{840} t^7 \right) + 3h^3\left(\frac{1}{3} t^3 + \frac{1}{20} t^5 + \frac{1}{840} t^7 + \frac{1}{60480} t^9 \right) \right)$$

The following solution for $n = 6$ will be obtained,

$$u = x^2\left(t + (\frac{5}{6} h + \frac{5}{3} h^2 + \frac{5}{3} h^3 + \frac{5}{6} h^4 + \frac{1}{6} h^5)t^3 + (\frac{1}{6} h^3 + \frac{1}{12} h^2 + \frac{1}{8} h^4 + \frac{1}{30} h^5)t^5 \right.$$

$$\left. + (\frac{1}{504} h^3 + \frac{1}{336} h^4 + \frac{1}{840} h^5)t^7 + \cdots \right)$$

To find a proper value of h the h-curves of $u_{tt}(0.5,0)$ given by the six terms of HAM approximation is drawn in Figure (2) that's shows the valid region of h is the interval $(-1.5,-0.5)$, so we chose $h = -0.9$. Tables (5 - 8) shows the absolute errors with six terms for x and t vary from 0.1 to 0.9, also the Maximum error for $x = 0.5$ showed in Figures (7 - 10).

Table (1): Absolute errors with six terms for Problem (1) by ADM

$t/x,y$	0.1	0.3	0.5	0.7
0.1	0	1.00000×10^{-9}	2.00000×10^{-9}	0
0.3	0	0	0	0
0.5	2.00000×10^{-9}	1.00000×10^{-9}	2.00000×10^{-9}	2.00000×10^{-9}
0.7	1.00000×10^{-9}	1.00000×10^{-9}	1.00000×10^{-9}	2.00000×10^{-9}
0.9	1.00000×10^{-9}	2.00000×10^{-9}	2.00000×10^{-9}	0

Table (2): Absolute errors with six terms for Problem (1) by VIM

$t/x,y$	0.1	0.3	0.5	0.7
0.1	0	0	0	-1.00000×10^{-9}
0.3	1.00000×10^{-9}	1.00000×10^{-9}	1.00000×10^{-9}	1.00000×10^{-9}
0.5	0	-1.00000×10^{-9}	-1.00000×10^{-9}	-1.00000×10^{-9}
0.7	0	-1.00000×10^{-9}	-2.00000×10^{-9}	-2.00000×10^{-9}
0.9	1.00000×10^{-9}	1.00000×10^{-9}	1.00000×10^{-9}	2.00000×10^{-9}

Table (3): Absolute errors with six terms for Problem (1) by

$t/x,y$	0.1	0.3	0.5	0.7
0.1	0	0	0	0
0.3	0	0	0	0
0.5	0	0	0	0
0.7	0	0	0	0
0.9	1.00000×10^{-9}	2.00000×10^{-9}	2.00000×10^{-9}	2.00000×10^{-9}

Table (4): Absolute errors with six terms for Problem (1) by

$t/x,y$	0.1	0.3	0.5	0.7
0.1	5.00000×10^{-8}	5.50000×10^{-8}	6.40000×10^{-8}	8.20000×10^{-8}
0.3	3.42000×10^{-7}	3.71000×10^{-7}	4.35000×10^{-7}	5.53000×10^{-7}
0.5	3.35000×10^{-7}	3.63000×10^{-7}	4.27000×10^{-7}	5.42000×10^{-7}
0.7	8.16000×10^{-7}	8.84000×10^{-7}	1.03700×10^{-6}	1.31900×10^{-6}
0.9	3.39200×10^{-6}	3.67500×10^{-6}	4.31200×10^{-6}	5.48100×10^{-6}

Table (5): Absolute errors with six terms for Problem (2) by ADM

t/x	0.1	0.3	0.5	0.7
0.1	1.00000×10^{-13}	0	0	1.00000×10^{-11}
0.3	0	1.00000×10^{-11}	2.00000×10^{-11}	1.00000×10^{-10}
0.5	1.00000×10^{-12}	0	1.00000×10^{-10}	1.00000×10^{-10}
0.7	0	1.00000×10^{-11}	0	0
0.9	1.00000×10^{-12}	0	0	1.00000×10^{-10}

Table (6): Absolute errors with six terms for Problem (2) by VIM

t/x	0.1	0.3	0.5	0.7
0.1	0	0	0	0
0.3	0	0	1.00000×10^{-11}	0
0.5	0	0	-1.00000×10^{-10}	0
0.7	0	0	0	0
0.9	0	0	0	0

Table (7): Absolute errors with six terms for Problem (2) by HPM

t/x	0.1	0.3	0.5	0.7
0.1	0	0	0	0
0.3	0	0	1.00000×10^{-11}	0
0.5	0	0	-1.00000×10^{-10}	0
0.7	0	0	0	0
0.9	0	0	0	0

Table (8): Absolute errors with six terms for Problem (2) by HAM

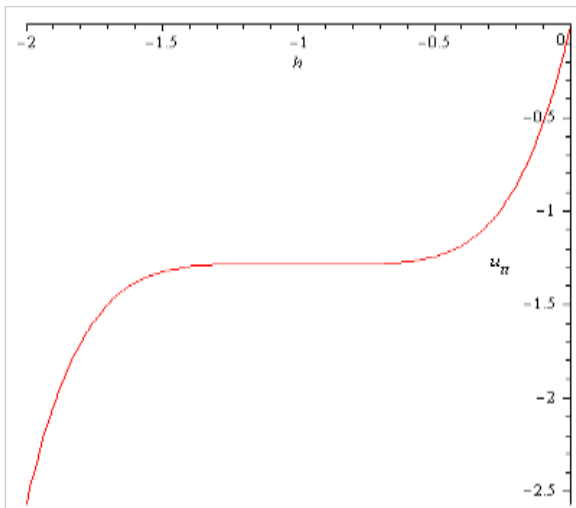| t/x	0.1	0.3	0.5	0.7
0.1	1.63000×10^{-11}	1.47000×10^{-10}	4.10000×10^{-10}	7.90000×10^{-10}
0.3	3.60000×10^{-10}	3.24000×10^{-9}	8.99000×10^{-9}	1.75000×10^{-8}
0.5	1.01400×10^{-9}	9.13000×10^{-9}	2.54000×10^{-8}	4.98000×10^{-8}
0.7	5.84000×10^{-10}	5.26000×10^{-9}	1.45000×10^{-8}	2.87000×10^{-8}
0.9	3.19800×10^{-9}	2.87900×10^{-8}	8.00000×10^{-8}	1.56700×10^{-7}

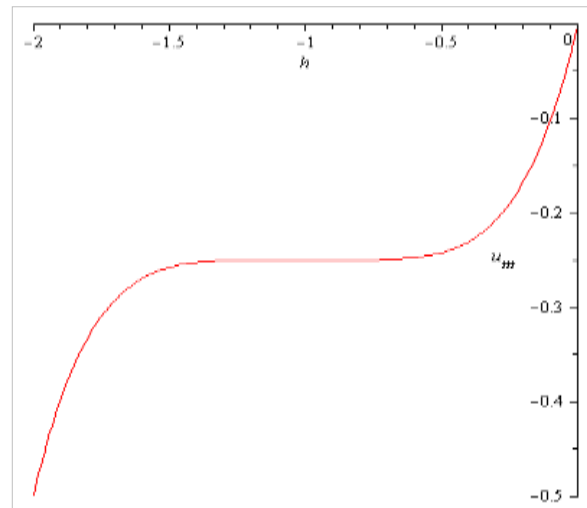Figure (1): The *h*-curve of $u_{tt}(0.5,0.5,0)$ based on the six terms of HAM.

Figure (2): The *h*-curve of $u_{ttt}(0.5,0)$ based on the six terms of HAM.
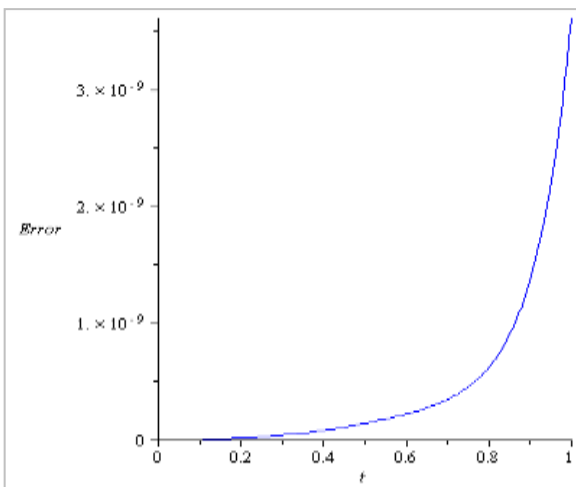

Figure (3): Errors between the solutions obtained using HPM with six terms and exact solution at x, $y = 0.5$
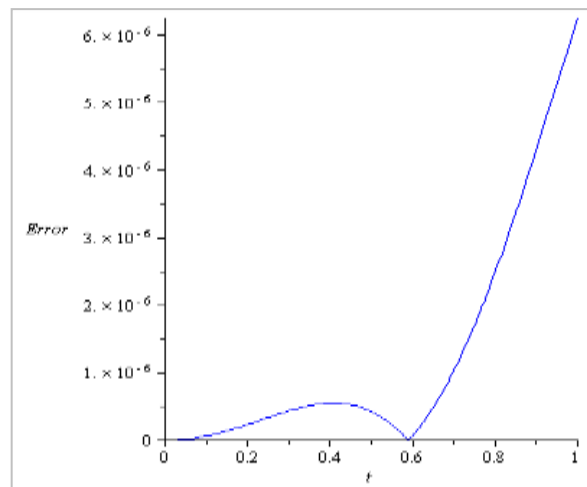

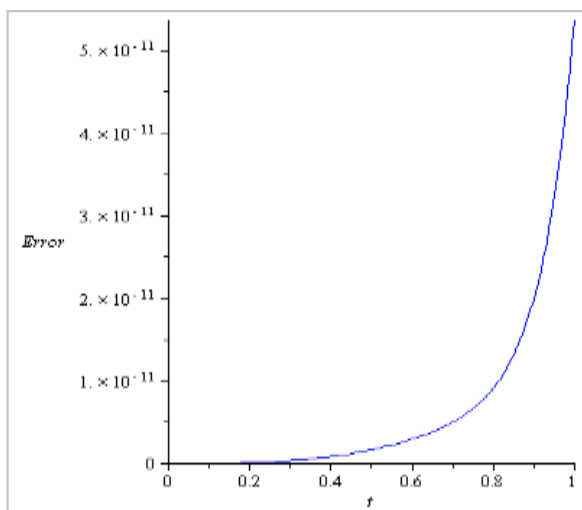Figure (4): Errors between the solutions obtained using HAM with six terms and exact solution at x, $y = 0.5$

Figure (5): Errors between the solutions obtained using ADM with six terms and exact solution at $x=0.5$
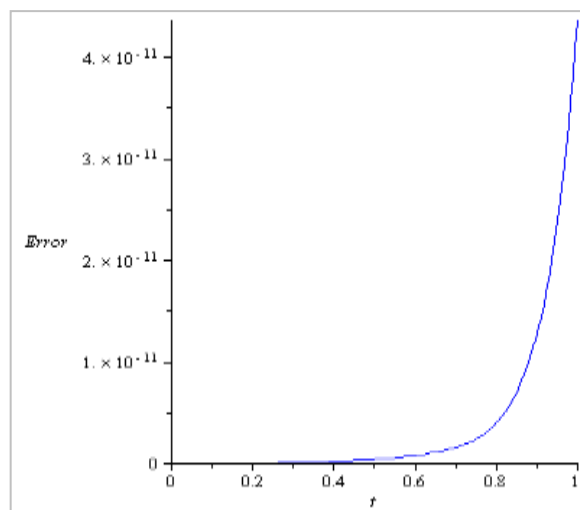

Figure (6): Errors between the solution obtained using VIM with six terms and exact solution at $x=0.5$
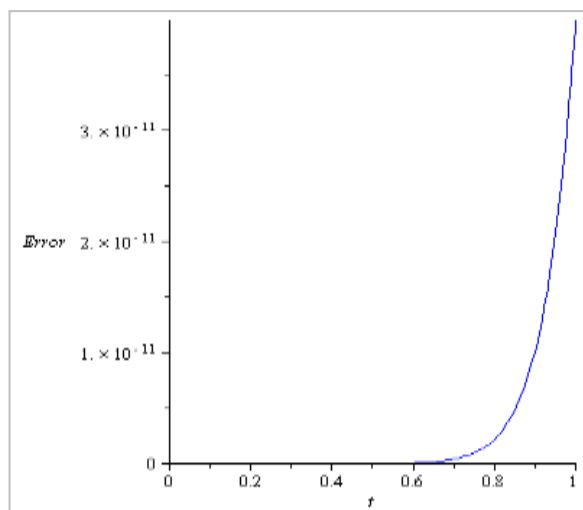

Figure (7): Errors between the solutions obtained using HPM with six terms and exact solution at $x=0.5$
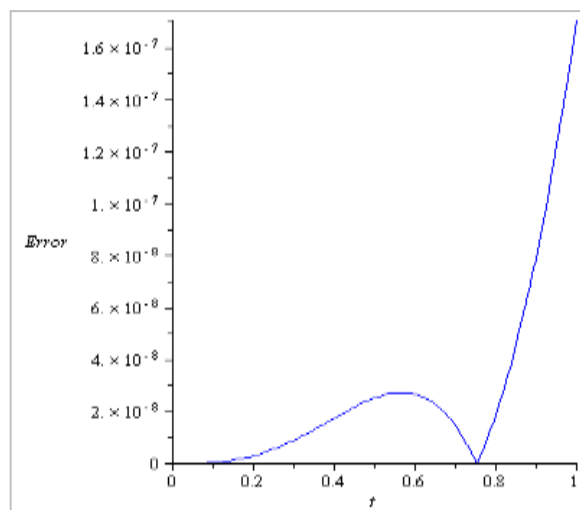

Figure (8): Errors between the solutions obtained using HAM with six terms and exact solution at $x=0.5$

6. Conclusions:

In this paper, we presented the application of the VIM, HPM and HAM in solving nonlinear wave-like Equations with variable coefficients. These methods was tested on two different examples where the numerical solutions clearly demonstrated that the VIM, HPM and HAM produces very accurate results which are very close to the exact solutions, also from tables (1-8) and figures (1-8) we conclude that HPM is

more accurate in applications than VIM, HAM and ADM. But HPM and VIM are easer in applications since ADM and HAM need the integrations in every iterations and the number of integrations are equal to the order of Equations, while The VIM and HPM deforms a difficult problem in to a simple problem which can be easily solved, also the VIM depends on the proper selection of the initial approximations $u_0(X,t)$.

Thus, these methods are capable of providing fast solutions that are highly accurate and reliable in solving this type of Equations.

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دراسة مقارنة طرائق شبه عددية لحل معادلات شبه الموجة اللاخطية بمعاملات متغيرة

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المستخلص

تناولنا في هذا البحث حل معادلة شبه الموجة بمعاملات متغيرة بثلاثة طرائق مختلفة وهي VIM , HPM و HAM. الحل الناتج من استعمال الطرائق الثلاث تمت مقارنته مع الحل التحليلي والحل الناتج من تطبيق طريقة ADM [8].